First simultaneous extraction of (spin) PDFs & FFs from global QCD analysis

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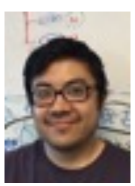

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JLab Angular Momentum Collaboration
http://www.jlab.org/JAM

- ***arXiv:1705.05889***
- ***PRD 94, 114004 (2016)***
- ***PRD 93, 074005 (2016)***

Outline

- Aim to understand quark & gluon spin structure of proton

- New Iterative Monte Carlo (IMC) methodology,
  with Bayesian determination of PDF errors
  $\longrightarrow$ single-fit technology problematic for some PDFs

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- First global QCD analysis of polarised inclusive DIS
  including (6 GeV) JLab data

- First MC extraction of fragmentation functions (FFs)
  from e^+e^- single-inclusive annihilation (SIA) data

- First combined analysis of polarised DIS + SIDIS + SIA data,
  with simultaneous extraction of PDFs and FFs
  $\longrightarrow$ resolves strange quark polarisation puzzle

Proton spin structure

- Question of how proton spin decomposed into its q & g constituents has engrossed community for ~ 30 years

$\longrightarrow$ in nonrelativistic quark model, spin of proton is carried entirely by quarks

$$\Delta\Sigma = \Delta u^+ + \Delta d^+ + \Delta s^+ = 1 \qquad\qquad \Delta q^+ \equiv \Delta q + \Delta\bar{q}$$

$\longrightarrow$ experimentally, from hadron weak decays have

$$\Delta u^+ - \Delta d^+ = g_A = 1.269(3) \qquad [\mathrm{SU}(2) \text{ symmetry}]$$

$$\Delta u^+ + \Delta d^+ - \Delta s^+ = a_8 = 0.586(31) \qquad [\mathrm{SU}(3) \text{ symmetry}]$$

$\longrightarrow$ third combination of flavours measured in polarised DIS

$$\int_0^1 dx\, g_1 = \frac{1}{18}\left(4\Delta u^+ + \Delta d^+ + \Delta s^+\right)$$

Proton spin structure

- Question of how proton spin decomposed into its q & g constituents has engrossed community for ~ 30 years

$\longrightarrow$ solving system of equations, early indications were that

$$\Delta\Sigma \approx 0 \; ! \qquad \Delta s^+ \approx -(0.1 - 0.2)$$

EMC (1988)

$\longrightarrow$ since proton spin sum requires

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

does remaining spin come from

— large gluon polarisation?

— orbital angular momentum?

$\longrightarrow$ stimulated many advances in theory, experiment & data analysis

Data analysis methodology

- Analysis of data requires estimating expectation values and variances of observables $\mathcal{O}$ (= PDFs, FFs)

$$E[\mathcal{O}] = \int d^n a \, \mathcal{P}(\vec{a}|\mathrm{data}) \, \mathcal{O}(\vec{a})$$

$$V[\mathcal{O}] = \int d^n a \, \mathcal{P}(\vec{a}|\mathrm{data}) \left[\mathcal{O}(\vec{a}) - E[\mathcal{O}]\right]^2$$

$\longrightarrow$ probability distribution

$$\mathcal{P}(\vec{a}|\mathrm{data}) \;\propto\; \mathcal{L}(\mathrm{data}|\vec{a}) \; \pi(\vec{a})$$

Bayes' theorem

priors

$\longrightarrow$ likelihood function

$$\mathcal{L}(\mathrm{data}|\vec{a}) \;\sim\; \exp\left[-\frac{1}{2}\,\chi^2(\vec{a})\right]$$

$$\chi^2(\vec{a}) = \sum_i \left(\frac{\mathrm{data}_i - \mathrm{theory}_i(\vec{a})}{\delta(\mathrm{data})}\right)^2$$

Data analysis methodology

- Standard method for evaluating E, V is "maximum likelihood"

$\longrightarrow$ maximize probability distribution

$$\mathcal{P}(\vec{a}|\mathrm{data}) \;\to\; \vec{a}_0$$

$\longrightarrow$ if $\mathcal{O}$ linear in parameters, and if probability is symmetric in all parameters

$$E[\mathcal{O}(\vec{a})] = \mathcal{O}(\vec{a}_0)\,, \qquad V[\mathcal{O}(\vec{a})] \to \mathrm{Hessian}$$

- In practice, since in general $E[f(\vec{a})] \neq f(E[\vec{a}])$, maximum likelihood method will sometimes fail

$\longrightarrow$ need more versatile approach ($e.g.$ Monte Carlo)

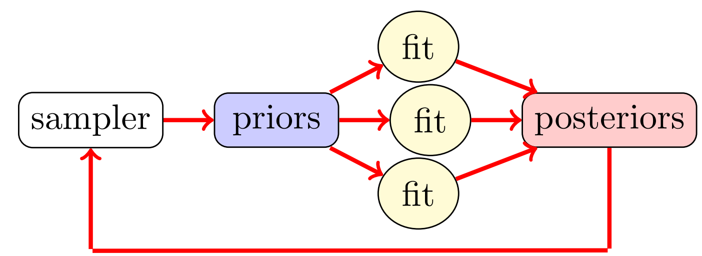
$$E[\mathcal{O}] \;\approx\; \frac{1}{N}\sum_k \mathcal{O}(\vec{a}_k)\,, \qquad V[\mathcal{O}] \;\approx\; \frac{1}{N}\sum_k \big[\mathcal{O}(\vec{a}_k) - E[\mathcal{O}]\big]^2$$

Data analysis methodology

- Use traditional functional form for input distribution shape

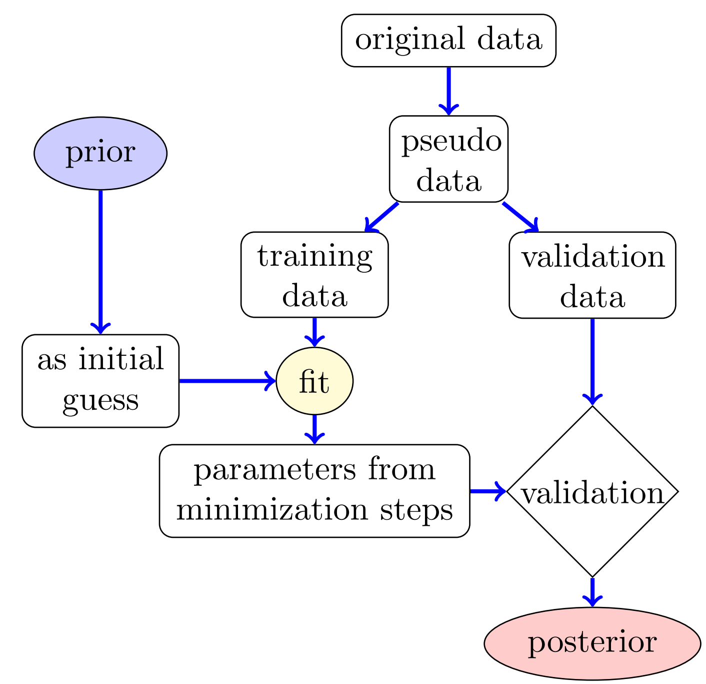
$$xf(x) = N x^{a} \, (1-x)^{b} \, (1 + c\sqrt{x} + d\, x + \cdots)$$

but sample significantly larger parameter space than possible in single-fit analyses



$\longrightarrow$ no assumptions for exponents

$\longrightarrow$ cross-validation to avoid overfitting



$\longrightarrow$ iterate until convergence criteria satisfied

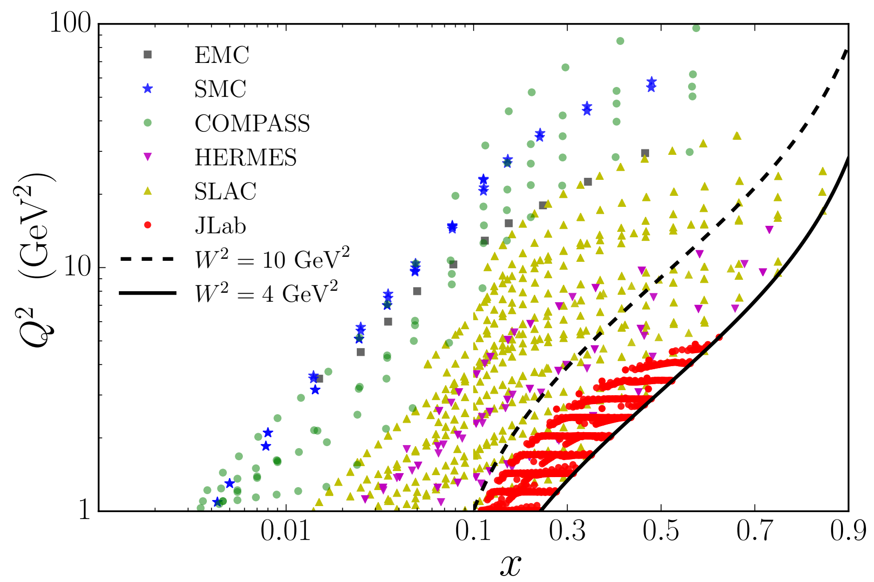
$\longrightarrow$ unambiguous determination of PDF uncertainties

Inclusive DIS global analysis

- Fit experimental asymmetries (longitudinal & transverse) rather than derived structure functions

$$A_\parallel = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} = D\,(A_1 + \eta\,A_2)$$

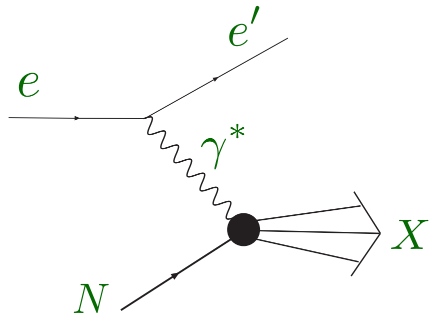
$$A_\perp = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\Leftarrow}} = d\,(A_2 - \zeta\,A_1)$$



where $$A_1 = (g_1 - \gamma^2 g_2)\frac{2x}{(1+\gamma^2)F_2 - F_L}$$

$$A_2 = \gamma(g_1 + g_2)\frac{2x}{(1+\gamma^2)F_2 - F_L}$$

$$\gamma^2 = \frac{4M^2x^2}{Q^2} \qquad x = \frac{Q^2}{2M(E_i - E_f)} \qquad F_2 \overset{\text{LO}}{=} \sum_q e_q^2\, x(q+\bar{q})$$



$\longrightarrow$ remove assumptions about $R = \sigma_L/\sigma_T$ ratio

Inclusive DIS global analysis

- Inclusive DIS data constrain Δu^+ & Δd^+ distributions
  $\longrightarrow$ mostly insensitive to polarised strangeness and glue

- Assume g_1, g_2 can be described as sum of twist $\tau = 2$ and higher twist (generally $1/Q$ suppressed) terms

$$g_1 = g_1^{\tau 2(\mathrm{TMC})} + g_1^{\tau 3(\mathrm{TMC})} + g_1^{\tau 4}$$

$$g_2 = g_2^{\tau 2(\mathrm{TMC})} + g_2^{\tau 3(\mathrm{TMC})}$$

includes target mass corrections (TMCs), proportional to

$$\rho^2 = 1 + \frac{4M^2x^2}{Q^2}$$

Inclusive DIS global analysis

- Leading twist contributions
  e.g. for proton (at leading order)

$$g^p_{1,\tau 2} \stackrel{\text{LO}}{=} \frac{1}{2}\Big(\frac{4}{9}\Delta u^+ + \frac{1}{9}\Delta d^+ + \frac{1}{9}\Delta s^+ + \cdots\Big)$$

- Work in moment space (faster numerical computations)

$\longrightarrow$ n-th moment of function $\Delta q^{(n)} \equiv \int_0^1 dx\, x^{n-1} \Delta q(x)$

- Structure function moments at next-to-leading order (NLO)

$$g^{(n)}_{1,\tau 2} = \frac{1}{2}\sum_q e_q^2 \left(\Delta C^{(n)}_{qq}\, \Delta q^{(n)} + \Delta C^{(n)}_g\, \Delta g^{(n)}\right)$$

$$g^{(n)}_{2,\tau 2} = -\frac{n-1}{n} g^{(n)}_{1,\tau 2}$$

hard scattering
(Wilson) coefficients

Inclusive DIS global analysis

- **Higher twist corrections**

→ twist-3 part of g_1 related to twist-3 part of g_2

$$g_1^{\tau 3}(x) = (\rho^2 - 1)\left[g_2^{\tau 3}(x) - 2\int_x^1 \frac{dz}{z} g_2^{\tau 3}(z)\right]$$

→ twist-3 part of g_2 parametrised via twist-3 PDFs D_q

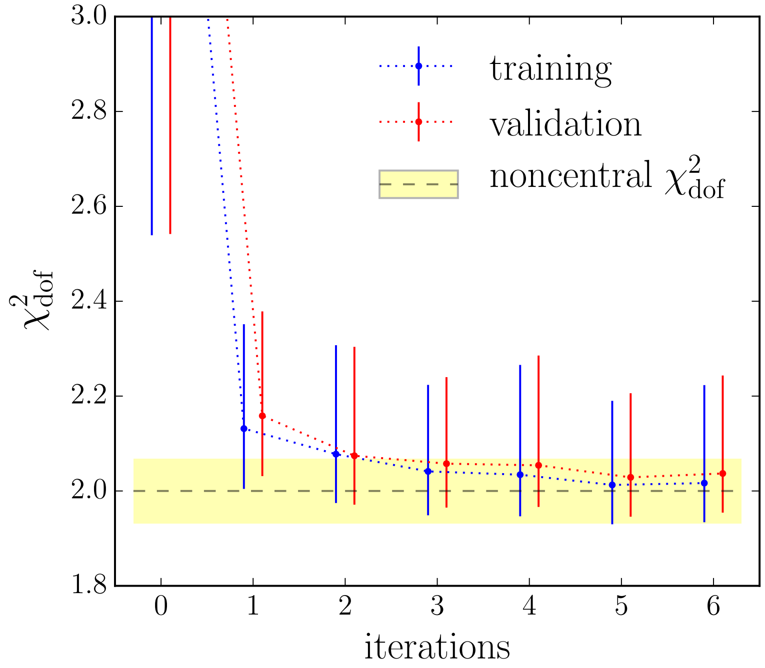
$$g_2^{\tau 3}(x) = D(x) - \int_x^1 \frac{dz}{z} D(z), \qquad D(x) = \sum_q e_q^2 \, D_q(x)$$

→ phenomenological twist-4 part

$$g_1^{\tau 4}(x) = \frac{H(x)}{Q^2}$$

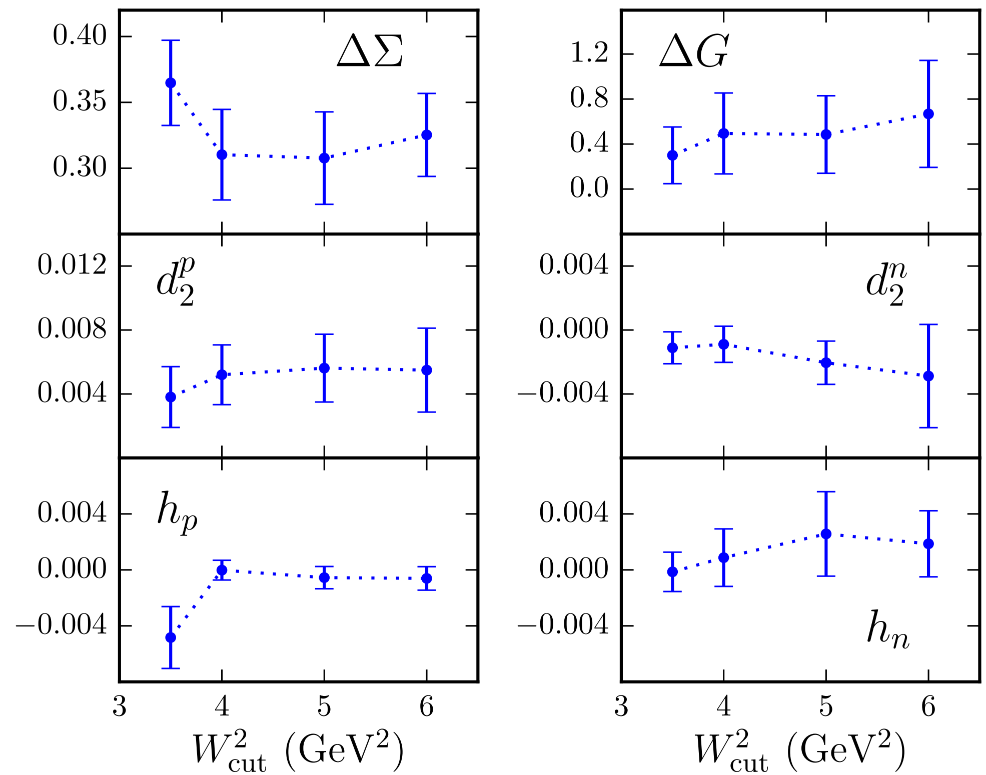
Inclusive DIS global analysis

- **Convergence criteria**



→ convergence after ~ 5–6 iterations

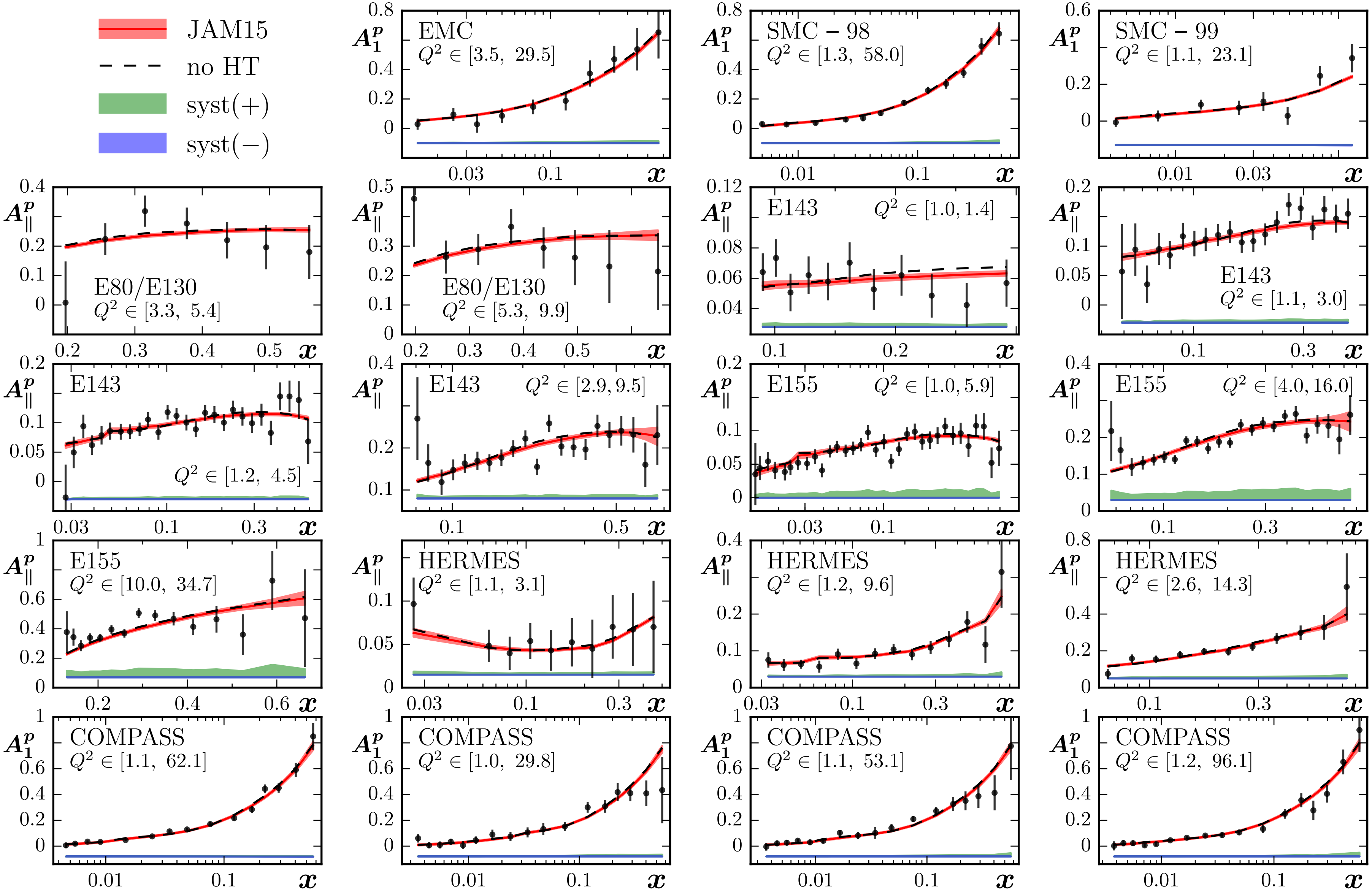
- **Sensitivity to kinematic cuts**



→ stability for $W^2 > 4$ GeV^2 and $Q^2 > 1$ GeV^2

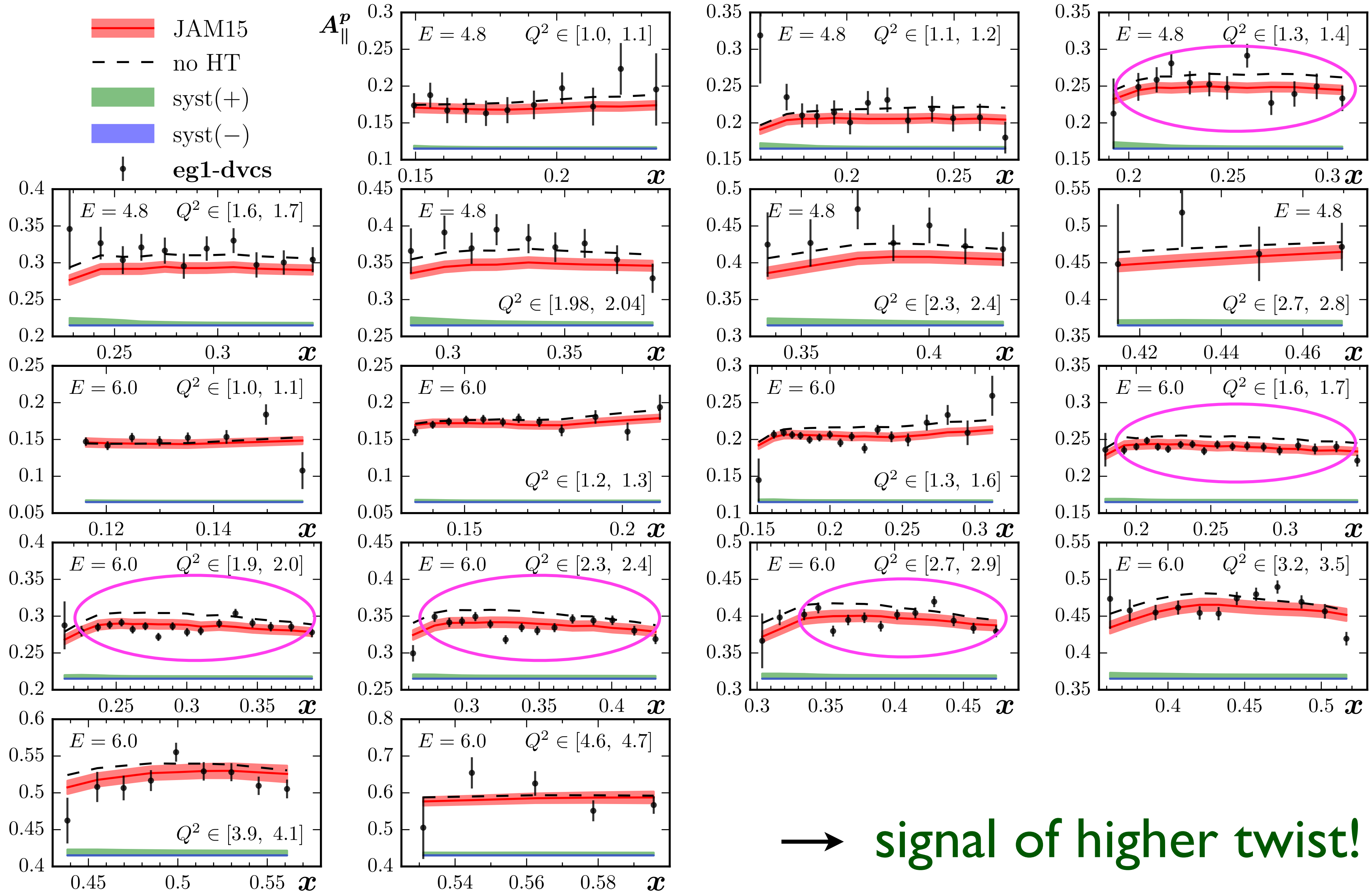
Inclusive DIS global analysis

- Previous world data



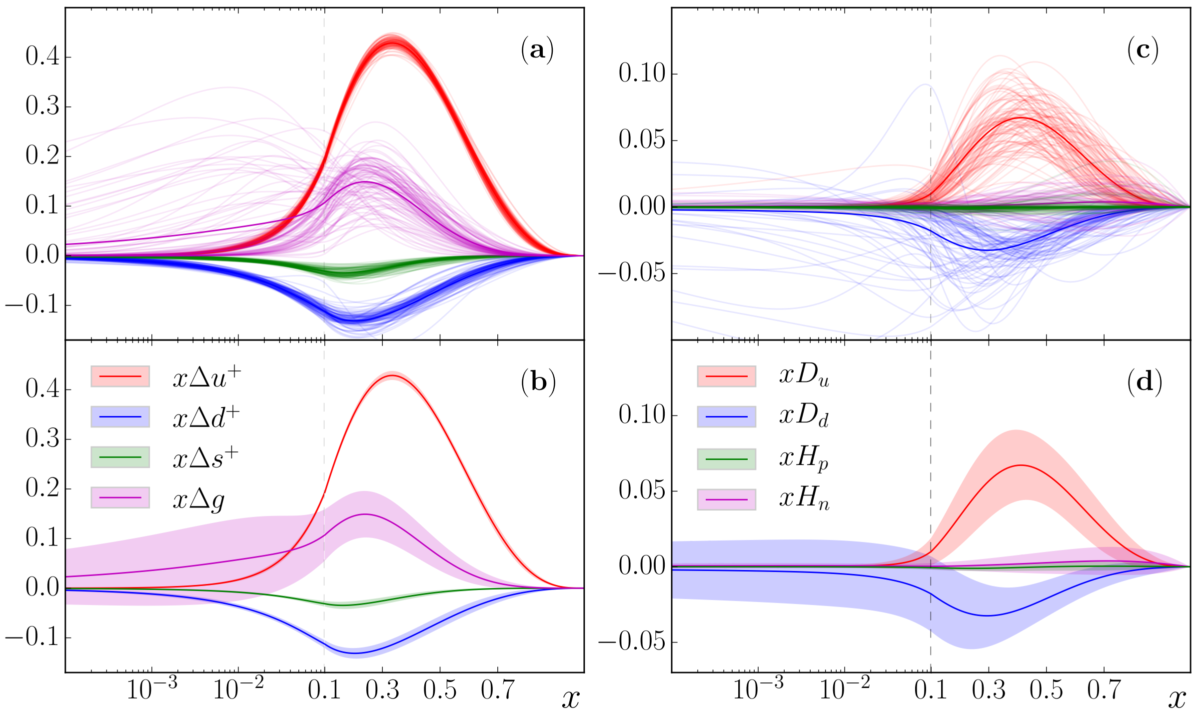
Impact of JLab data

- JLab eg1-dvcs (CLAS) data



→ signal of higher twist!

Impact of JLab data



→ twist-3 PDFs large!

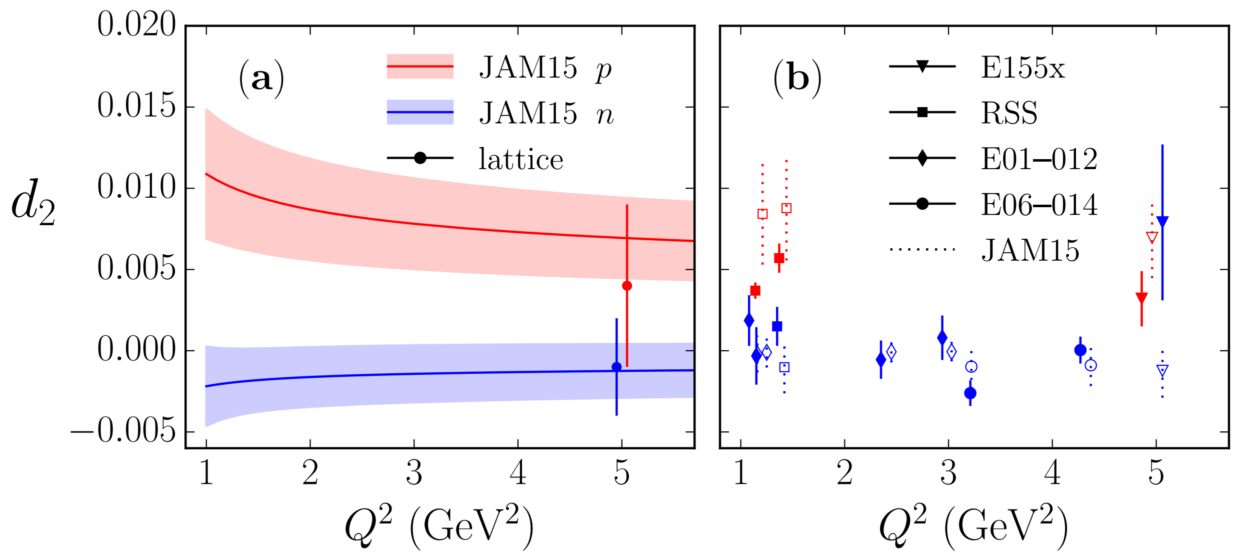
→ same sign as twist-2

→ twist-4 term negligible

→ matrix element

$$d_2 = 2g_1^{(3)} + 3g_2^{(3)}$$

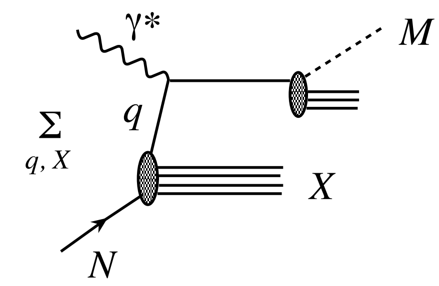
related to
"*color polarisability*"
or "*transverse force*"
acting on quarks



Polarization of quark sea?

- Inclusive DIS data cannot distinguish between q and $\bar{q}$

  $\longrightarrow$ semi-inclusive DIS sensitive to Δq & $\Delta\bar{q}$



$$\sim \sum_q e_q^2 \left[\Delta q(x)\, D_q^h(z) + \Delta\bar{q}(x)\, D_{\bar{q}}^h(z)\right]$$

$$z = \frac{E_M}{E_{\gamma^*}}$$

  $\longrightarrow$ but need fragmentation functions!

- Global analysis of DIS + SIDIS data gives different *sign* for strange quark polarisation for different fragmentation functions!

  $\longrightarrow$ $\Delta s > 0$ for "DSS" parametrisation *de Florian et al., PRD75, 094009 (2007)*

  $\Delta s < 0$ for "HKNS" parametrisation *Hirai et al., PRD75, 114010 (2007)*

  $\longrightarrow$ need to understand origin of differences in fragmentation!

IMC analysis of fragmentation functions

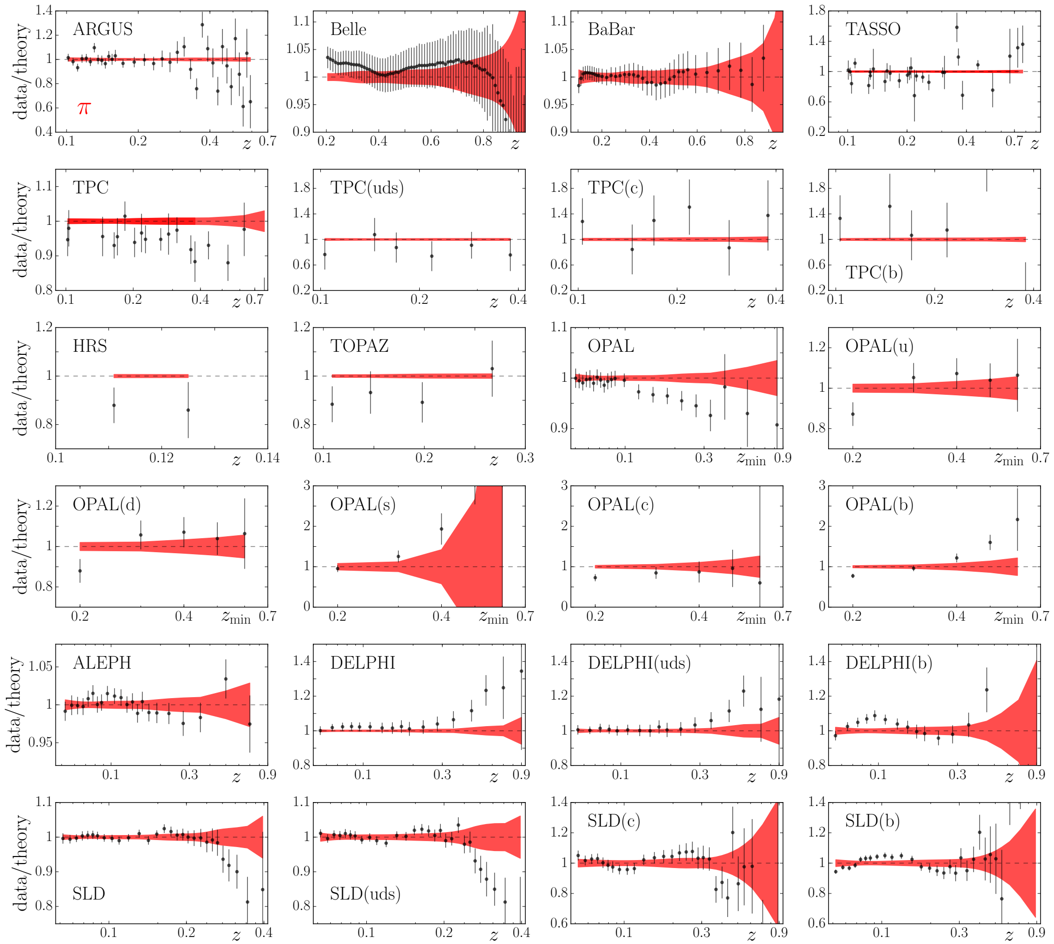
- Analyse single-inclusive e^+e^- annihilation data for pion & kaon production from DESY, CERN, SLAC & KEK from $Q \sim 10$ GeV to Z-boson pole

prompt (conventional)

Experiment	Ref.	Observable	Q (GeV)	Pions $N_{\rm dat}$	Pions norm.	Pions χ^2	Kaons $N_{\rm dat}$	Kaons norm.	Kaons χ^2
ARGUS	[26]	Inclusive	9.98	35	1.024 (1.058)	51.1 (55.8)	15	1.007	8.5
Belle	[38,39]	Inclusive	10.52	78	0.900 (0.919)	37.6 (21.7)	78	0.988	10.9
BABAR	[40]	Inclusive	10.54	39	0.993 (0.948)	31.6 (70.7)	30	0.992	4.9
TASSO	[23–25]	Inclusive	12-44	29	(*)	37.0 (38.8)	18	(*)	14.3
TPC	[27–29]	Inclusive	29.00	18	1	36.3 (57.8)	16	1	47.8
		uds tag	29.00	6	1	3.7 (4.6)			
		b tag	29.00	6	1	8.7 (8.6)			
		c tag	29.00	6	1	3.3 (3.0)			
HRS	[30]	Inclusive	29.00	2	1	4.2 (6.2)	3	1	0.3
TOPAZ	[37]	Inclusive	58.00	4	1	4.8 (6.3)	3	1	0.9
OPAL	[32,33]	Inclusive	91.20	22	1	33.3 (37.2)	10	1	6.3
		u tag	91.20	5	1.203 (1.203)	6.6 (8.1)	5	1.185	2.1
		d tag	91.20	5	1.204 (1.203)	6.1 (7.6)	5	1.075	0.6
		s tag	91.20	5	1.126 (1.200)	14.4 (11.0)	5	1.173	1.5
		c tag	91.20	5	1.174 (1.323)	10.7 (6.1)	5	1.169	13.2
		b tag	91.20	5	1.218 (1.209)	34.2 (36.6)	4	1.177	10.9
ALEPH	[34]	Inclusive	91.20	22	0.987 (0.989)	15.6 (20.4)	18	1.008	6.1
DELPHI	[35,36]	Inclusive	91.20	17	1	21.0 (20.2)	27	1	3.9
		uds tag	91.20	17	1	13.3 (13.4)	17	1	22.5
		b tag	91.20	17	1	41.9 (42.9)	17	1	9.1
SLD	[31]	Inclusive	91.28	29	1.002 (1.004)	27.3 (36.3)	29	0.994	14.3
		uds tag	91.28	29	1.003 (1.004)	51.7 (55.6)	29	0.994	42.6
		c tag	91.28	29	0.998 (1.001)	30.2 (40.4)	29	1.000	31.7
		b tag	91.28	29	1.005 (1.005)	74.6 (61.9)	28	0.992	134.1
Total:				459		599.3 (671.2)	391		395.0

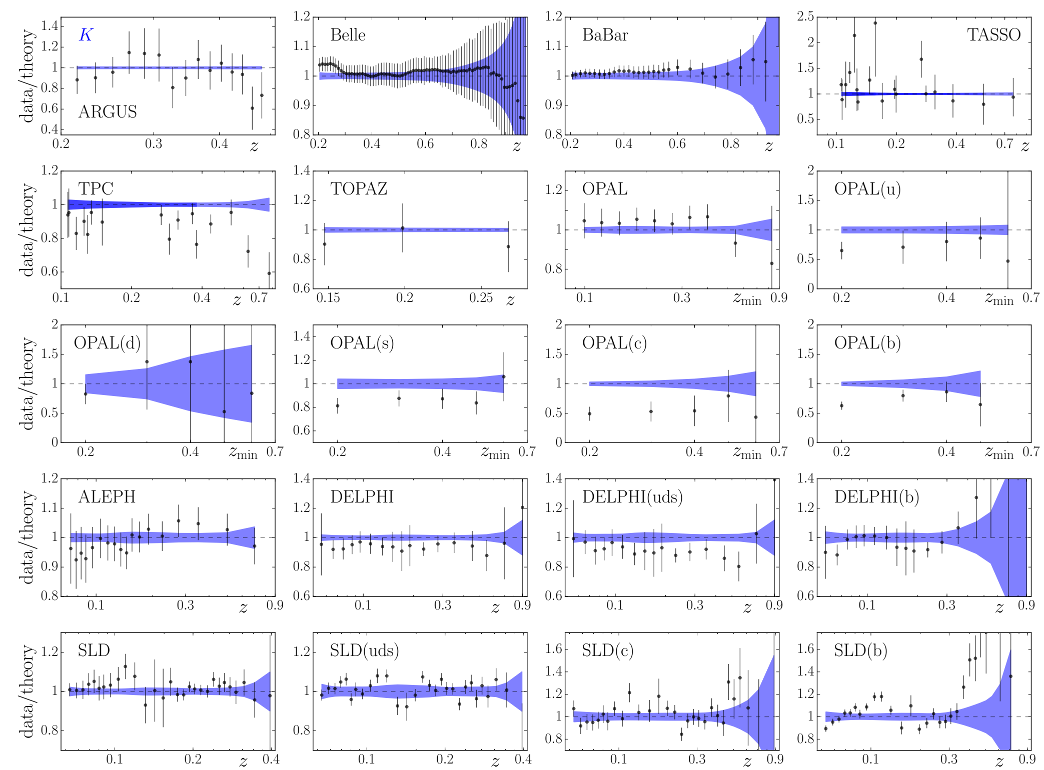
$\boxed{\chi^2/N_{\rm dat} = 1.31\ (1.46) \qquad \chi^2/N_{\rm dat} = 1.01}$

IMC analysis of fragmentation functions



pion production

kaon production

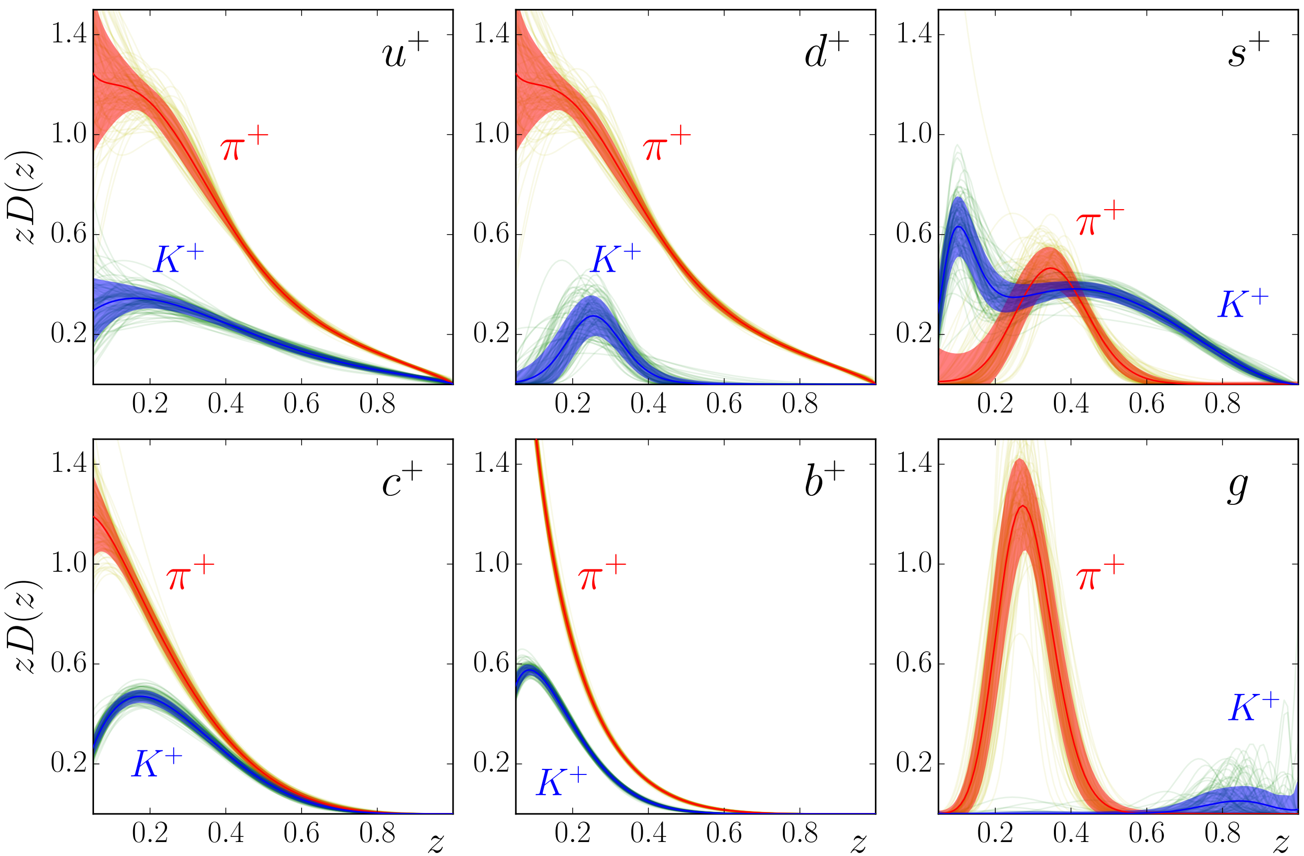


$\longrightarrow$ generally good agreement

$\longrightarrow$ some issues at high z

$\longrightarrow$ smaller χ^2 for K *cf.* π
due to larger uncertainties

IMC analysis of fragmentation functions

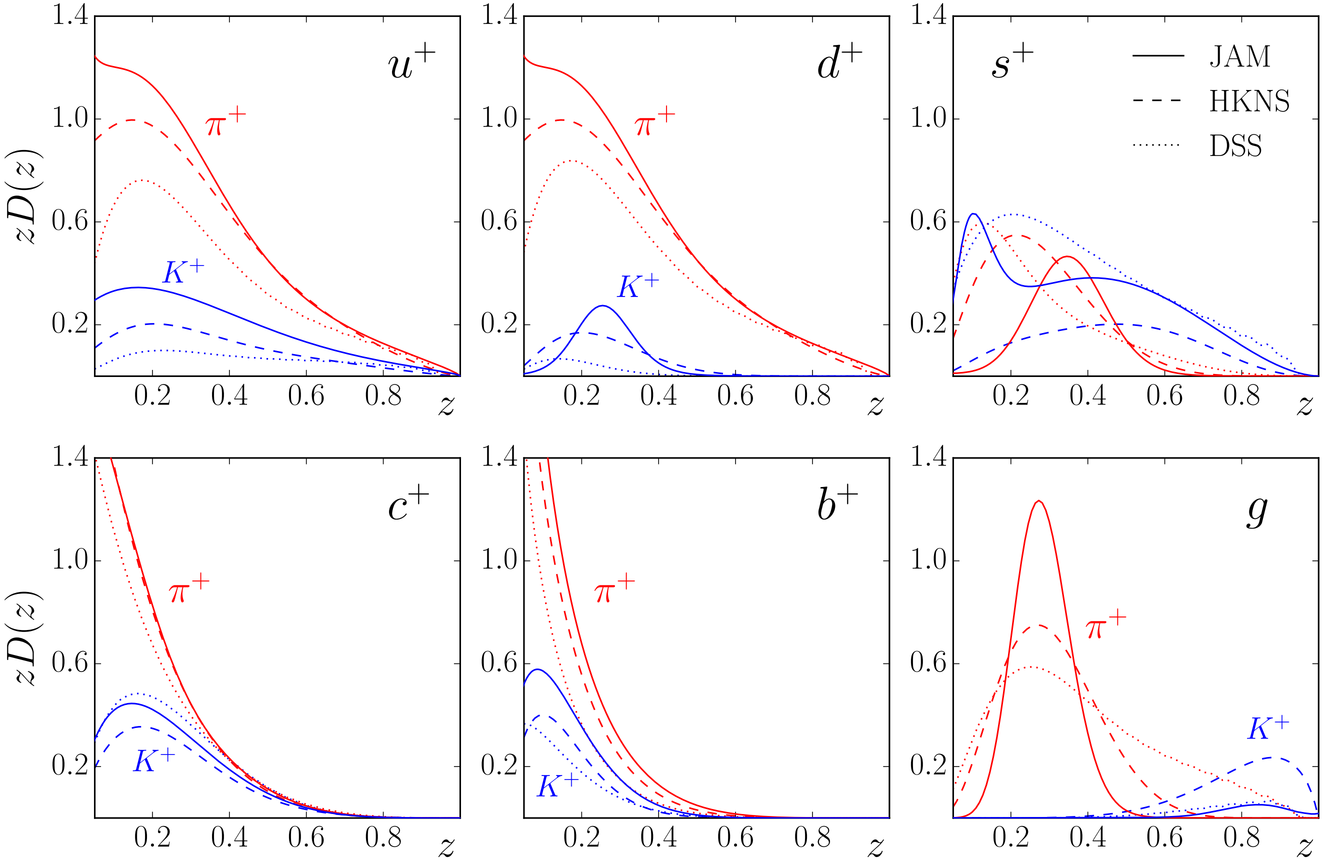


→ favoured FFs well constrained; unfavoured not as well…

→ nontrivial shape of $s \to K$ fragmentation

→ very hard $g \to K$ fragmentation? (robust feature!)

IMC analysis of fragmentation functions

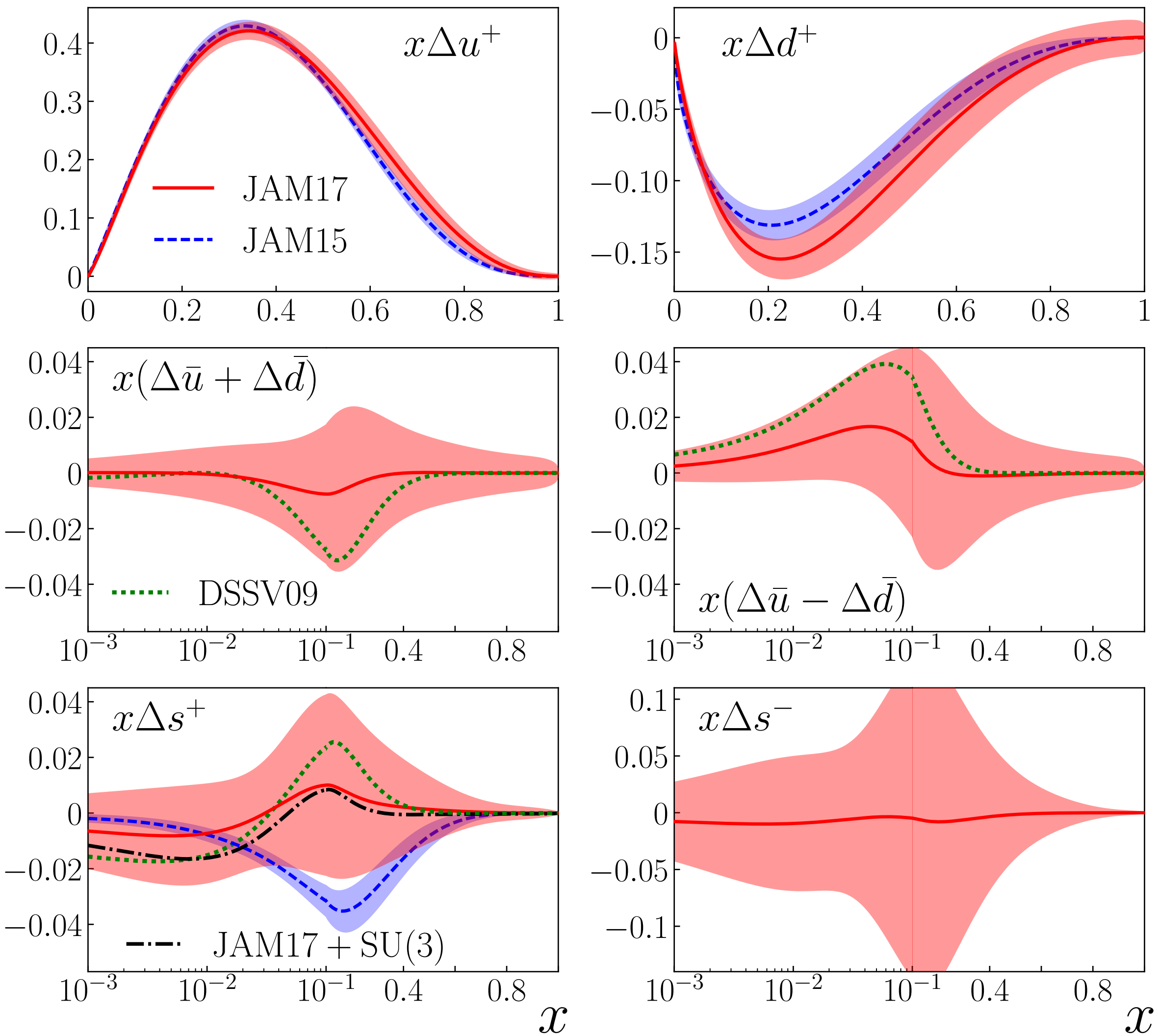


→ qualitatively similar behavior as in previous analyses

→ JAM strange FF closer to DSS parametrization
— impact on Δs^+ extraction??

Simultaneous analysis

- Using same IMC tools, new analysis attempted first ever simultaneous determination of spin PDFs and FFs, fitting to DIS, SIA and polarised SIDIS (HERMES, COMPASS) data



→ no assumption of SU(3) symmetry

→ Δs slightly > 0 at high x, consistent with zero

→ $\Delta s - \Delta\bar{s}$ & $\Delta\bar{u} - \Delta\bar{d}$ consistent with zero

Simultaneous analysis

- Using same IMC tools, new analysis attempted first ever simultaneous determination of spin PDFs and FFs, fitting to DIS, SIA and polarised SIDIS (HERMES, COMPASS) data

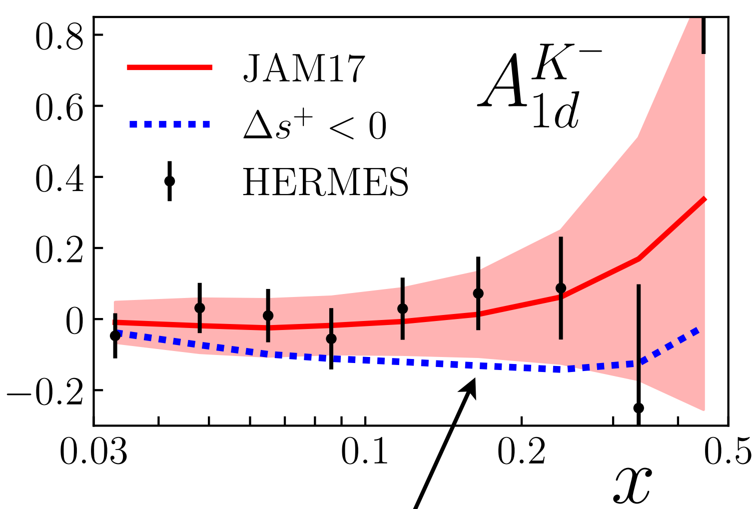
process	target	$N_{\rm dat}$	χ^2
DIS	$p, d, {}^3$He	854	854.8
SIA $(\pi^\pm, K^\pm)$		850	997.1
SIDIS $(\pi^\pm)$			
HERMES [15]	d	18	28.1
HERMES [15]	p	18	14.2
COMPASS [16]	d	20	8.0
COMPASS [17]	p	24	18.2
SIDIS $(K^\pm)$			
HERMES [15]	d	27	18.3
COMPASS [16]	d	20	18.7
COMPASS [17]	p	24	12.3
Total:		**1855**	**1969.7**

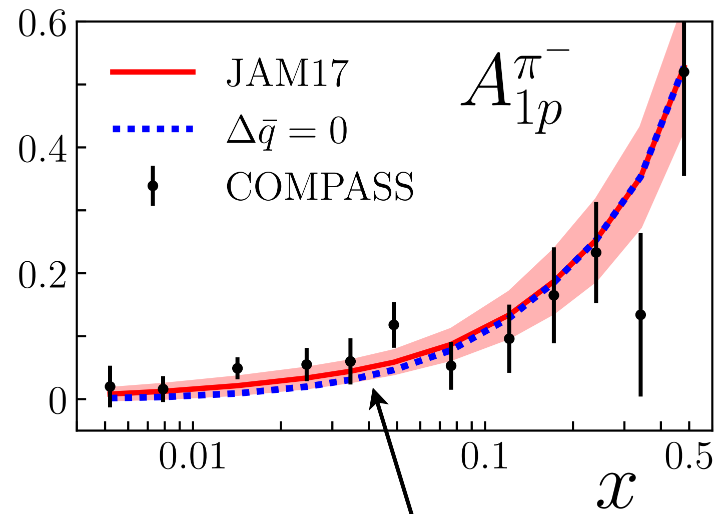
$\longrightarrow$ SIDIS data dwarfed by DIS & SIA

$\longrightarrow$ smaller χ^2 for K *cf.* π SIDIS due to larger errors

Simultaneous analysis

- Slightly positive Δs^+ at $x \sim 0.1$ attributed directly to deuteron K^- production data



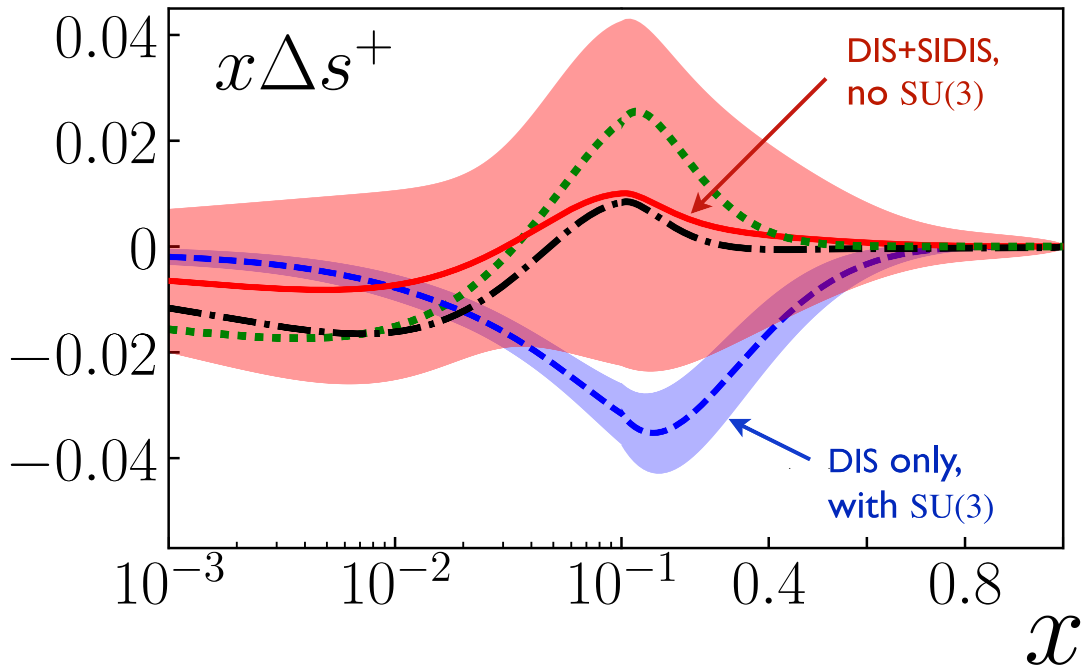


significantly worse fit using negative JAM15 Δs^+ (also for COMPASS and p data)

no such effect observed for pion data sensitive to $\Delta \bar{u}$ & $\Delta \bar{d}$

Simultaneous analysis

- Polarised strangeness in previous, DIS-only analyses (*e.g.* JAM15) was negative at $x \sim 0.1$, induced by SU(3) and parametrisation bias

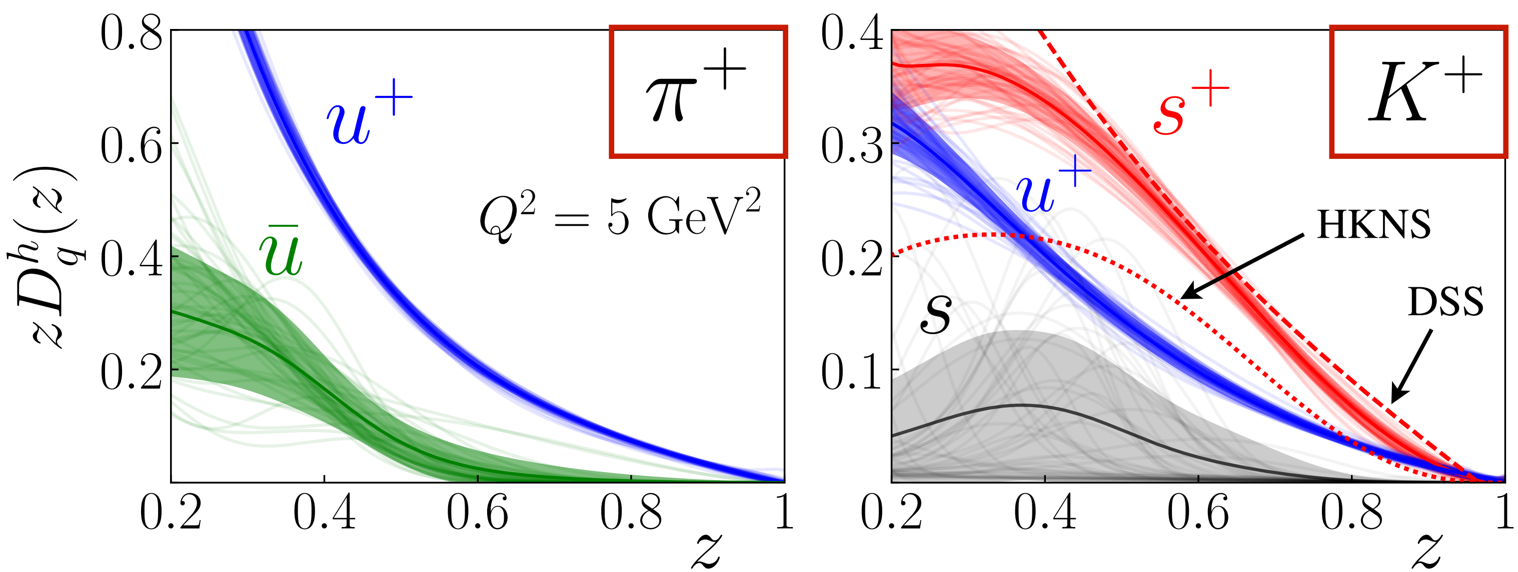


→ negligible sensitivity to Δs^+ from DIS data & evolution
— SU(3) pulls Δs^+ to generate moment ~ -0.1

→ negative peak at $x \sim 0.1$ induced by fixing b parameter to $\sim 6 - 8$

Simultaneous analysis

- Fragmentation functions similar to previous SIA-only analysis



→ some constraint from SIDIS on unfavoured FFs $(e.g.\ \ s \to K^+)$ but uncertainties still large

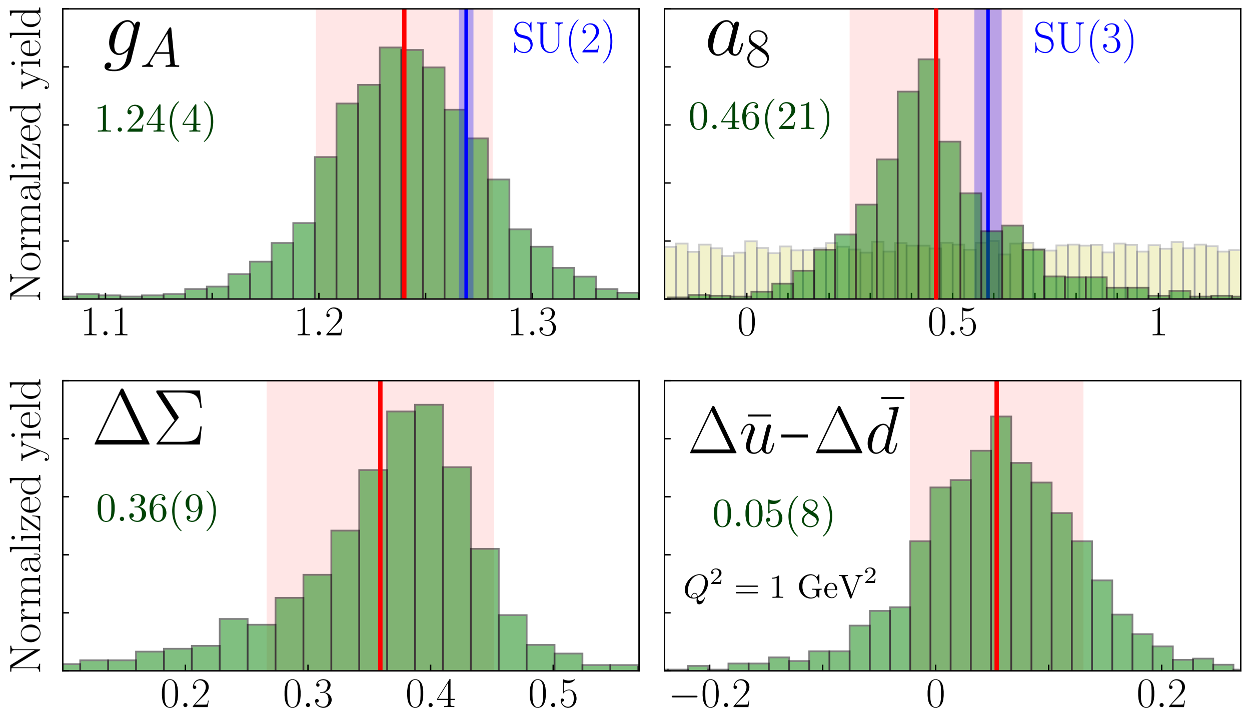
→ new result more consistent with DSS at moderate z

→ qualitatively similar to NJL-Jet model calculation

Matevosyan et al., PRD ***83****, 114010 (2011)*

Simultaneous analysis

- Statistical distribution of lowest moments (axial charges)



$\longrightarrow$ triplet charge g_A consistent with $\mathrm{SU(2)}$ value

$\longrightarrow$ hint of $\mathrm{SU(3)}$ breaking in octet charge a_8 *cf. Bass, Thomas, PLB **684**, 216 (2010)*

$\longrightarrow$ less negative $\Delta s = -0.03(10)$ gives larger total helicity $\Delta\Sigma = 0.36(9)$

Outlook

- New paradigm in global QCD analysis
  — simultaneous determination of collinear distributions
  using Monte Carlo sampling of parameter space

- First combined analysis of polarised DIS + SIDIS + SIA,
  with consistent extraction of spin PDFs and FFs
  without assuming $\mathrm{SU}(3)$ symmetry
  $\longrightarrow$ resolves strange quark polarization puzzle: $\Delta s = -0.03(10)$
  (less negative than traditional $\mathrm{SU}(3)$ value $\Delta s \sim -0.1$)

- Near–term future: "universal" QCD analysis of all observables
  sensitive to collinear (unpolarised & polarised) PDFs and FFs

- Longer–term: apply IMC technology to global QCD analysis
  of transverse momentum dependent (TMD) PDFs and FFs