Theoretical challenges in semi-inclusive DIS

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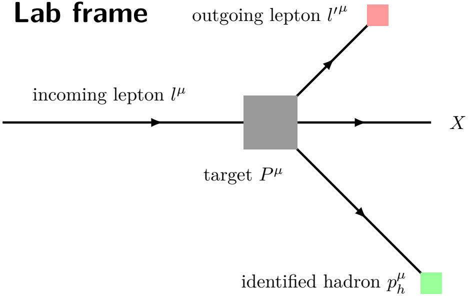
Outline

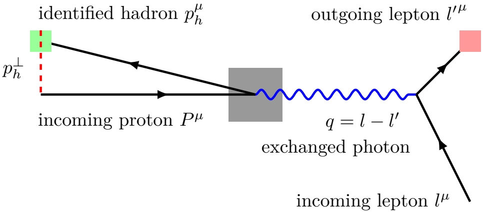
- Basic theory aspects of SIDIS
- Highlight existing issues of the framework
- Highlights recent progress

Semi inclusive deep inelastic scattering (SIDIS)

Lab frame



Breit frame

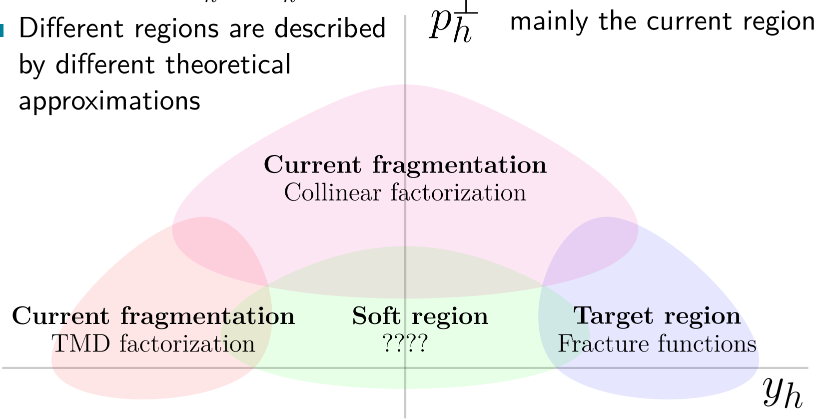


- Process is dominated by one photon exchange with large virtuality $Q^2 \gg \lambda_{\mathrm{QCD}}$
- In the Breit frame the proton and the photon has zero transverse momentum.
- The detected hadron has $p_h^{\perp}$ relative to proton-photon axis
- **Key question** : How is $p_h^{\perp}$ generated at short distances?

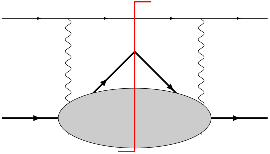
Kinematic regions

- Detected hadron's rapidity $y_h = \frac{1}{2} \ln \left(\frac{p_h^+}{p_h^-} \right)$
- Current region $p_h^- \gg p_h^+$
- Target region $p_h^- \ll p_h^+$
- Different regions are described by different theoretical approximations

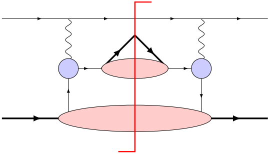
- The higher the c.o.m energy, the larger the separation among the regions
- In this talk I will discuss mainly the current regions



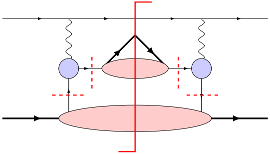
Factorization in the current region



Full contrubuion

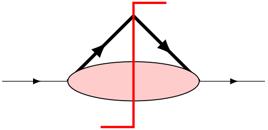


Leading regions: Hard and soft blobs

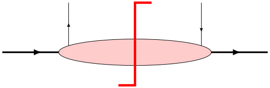


Kinematic approximations to separate the soft blobs from hard blob

- Each blob represents all possible Feynman graphs with restricted power counting for their internal propagators
- The soft parts can have a physical interpretation

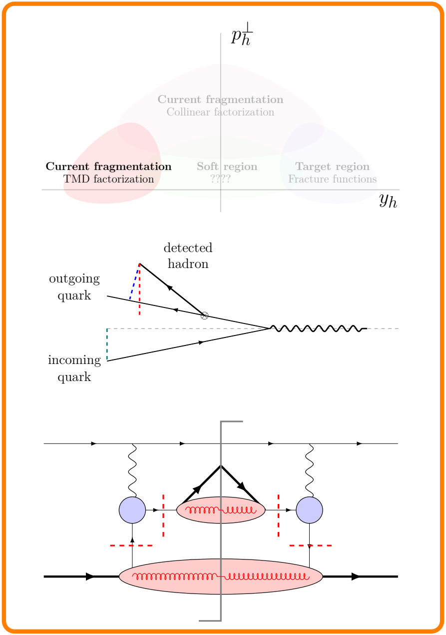


→ parton to hadron fragmentation functions (FFs)

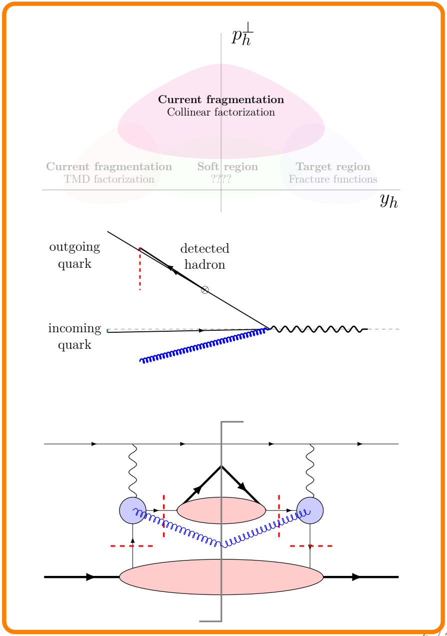


→ parton in hadron distribution functions (PDFs)

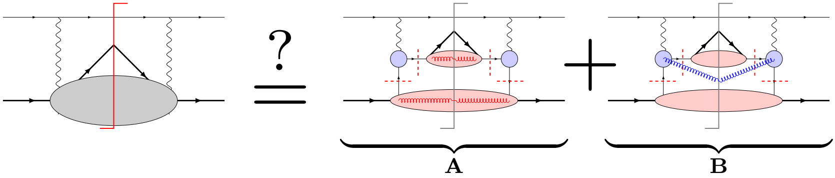
Small transverse momentum



Large transverse momentum



Combining large and small $p_h^\perp$ approximation



- $\mathbf{A}$ is only optimal for **small** $p_h^\perp$ and its accuracy degrades for **large** $p_h^\perp$
- $\mathbf{B}$ is only optimal for **large** $p_h^\perp$ and its accuracy degrades for **small** $p_h^\perp$
- The $p_h^\perp$ regions where the accuracy of $\mathbf{A}$ and $\mathbf{B}$ degrade, their contributions needs to be suppressed
- The two approximations necessarily overlaps in regions where $p_h^\perp$ is neither small or large. One needs to avoid the double counting
- The latter two issues are resolved by using a **substraction** method known as $W + Y$

The subtraction method $W+Y$

- SIDIS reaction

$$l + P \to l' + p_h + X$$

- SIDIS invariants

$$q = l - l' \qquad Q^2 = -q^2$$

$$x = \frac{Q^2}{2P\cdot q} \qquad z = \frac{P\cdot p_h}{P\cdot q}$$

$$q_T = p_h^{\perp}/z$$

- SIDIS cross section

$$\Gamma \equiv \frac{d\sigma}{dx dQ^2 dz dq_T}$$

- The W+Y construction

$$\begin{aligned}\Gamma =&\Gamma \\ =&\mathbf{T}_{\rm TMD}\Gamma + [\Gamma - \mathbf{T}_{\rm TMD}\Gamma] \\ =&\underbrace{\mathbf{T}_{\rm TMD}\Gamma}_{\mathbf{W}} + \underbrace{\mathbf{T}_{\rm coll}\,[\Gamma - \mathbf{T}_{\rm TMD}\Gamma]}_{\mathbf{Y}} \\ &+ \mathcal{O}(m^2/Q^2)\Gamma\end{aligned}$$

- Nomenclature

$$\mathrm{W} \equiv \mathbf{T}_{\rm TMD}\Gamma$$

$$\mathrm{FO} \equiv \mathbf{T}_{\rm coll}\Gamma$$

$$\mathrm{ASY} \equiv \mathbf{T}_{\rm coll}\mathbf{T}_{\rm TMD}\Gamma$$

$$\mathrm{Y} \equiv \mathrm{FO} - \mathrm{ASY}$$

Example of the subtraction method

Moffat, Melnitchouk, Rogers, NS 2017 (PRD.95.096008)

- Consider inclusive DIS $(l + P \to l' + X)$ in scalar diquark model with

$$\mathcal{L}_{\text{int}} = -\lambda \bar{\Psi}_N \psi_q \phi + \text{H.C.}$$

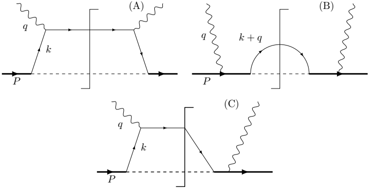
$$E' \frac{d\sigma}{d^3 l'} = \frac{\alpha^2 L_{\mu\nu} W^{\mu\nu}}{2\pi(s - M^2) Q^4}$$

- $L_{\mu\nu}$ is the leptonic tensor and $W^{\mu\nu}$ is the hadronic tensor

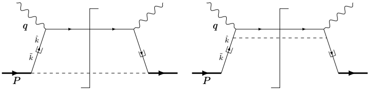
$$W^{\mu\nu} = \sum_i C_i^{\mu\nu} F_i \left(x, Q^2\right)$$

$$F_i \left(x, Q^2\right) = \int \frac{\mathrm{d}^2 \boldsymbol{k}_\mathsf{T}}{(2\pi)^2} \mathcal{F}_i(x, Q^2, k_\mathrm{T}^2)$$

- Contributions at lowest order in λ

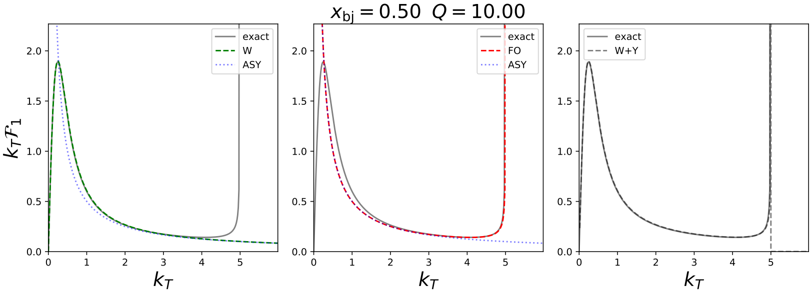


- Approximations



small k_T large k_T

Example of the subtraction method



- Exact and "FO" has an upper k_T limit and its singular behavior is due to the jacobian peak
- In contrast "W" and "ASY" has no upper k_T limit.
- The k_T integration of "W" and "ASY" have logaritmic divergence. They need to be renormalized individually.
- "FO" diverges as $k_T \to 0$. Its singularity is canceled by "ASY"
- A finite F_1 is given by $F_1 = \int \frac{d^2 \boldsymbol{k}_T}{(2\pi)^2} [\mathrm{W} - \mathrm{ASY} + \mathrm{FO}]$

Connection with collinear SIDIS

Collins, Gamberg, Prokudin, Rogers, NS, Wang 2016 (PRD94,034014)

- **TMD factorization**: designed to be an optimal calculation point-by-point in q_T

$$\frac{d\sigma^{\rm TMD}}{dxdQ^2dzdq_T} \equiv \Gamma = \underbrace{\mathbf{T}_{\rm TMD}\Gamma}_{\mathbf{W}} + \underbrace{\mathbf{T}_{\rm coll}\left[\Gamma - \mathbf{T}_{\rm TMD}\Gamma\right]}_{\mathbf{Y}} + \mathcal{O}(m^2/Q^2)\Gamma$$

- **Collinear factorization**: designed to describe the q_T integrated cross section

$$\frac{d\sigma^{\rm coll}}{dxdQ^2dz} = \frac{d\sigma^{\rm Born}}{dxdQ^2dz} + \frac{d\sigma^{\rm Virtual}}{dxdQ^2dz} + \frac{d\sigma^{\rm Real}}{dxdQ^2dz} - \frac{d\sigma^{\rm CC}}{dxdQ^2dz}$$

- Can we relate both formalism? e.g.

$$\frac{d\sigma^{\rm coll}}{dxdQ^2dz} \stackrel{?}{=} \int dq_T \frac{d\sigma^{\rm TMD}}{dxdQ^2dzdq_T}$$

$$\sum_q e_q^2\, f_1^q(x,Q^2) D_{h/q}(z,Q^2) \stackrel{?}{=} \int dq_T W(q_T)$$

Connection with collinear SIDIS

Collins, Gamberg, Prokudin, Rogers, NS, Wang 2016 (PRD94,034014)

$$\frac{d\sigma^{\text{TMD}}}{dx dQ^2 dz dq_T} \equiv \Gamma = \frac{H}{z^2} \int_0^\infty \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}}\, \widetilde{W}(b_*) + Y$$

$$\widetilde{W}(b_*) \equiv e^{-S_{\text{pert}}(Q,b_*) - S_{\text{NP}}^{\text{f}_1}(Q,b) - S_{\text{NP}}^{\text{D}_1}(Q,b)}\, \widetilde{F}(b_*)$$

$$\widetilde{F}(b_*) = \sum_q e_q^2 \left(\hat{C}^{\text{f}_1}_{q\leftarrow i} \otimes f_1^i(x,\mu_b)\right) \left(\hat{C}^{\text{D}_1}_{j\leftarrow q} \otimes D_{h/j}(z,\mu_b)\right)$$

$$b_*(b) = \sqrt{\frac{b^2}{1 + b^2/b_{\text{max}}^2}}$$

- Limiting behavior

$$\lim_{b\to 0} \widetilde{b}_*(b) = 0$$

$$\lim_{b\to 0} \widetilde{W}(b_*) = 0$$

$$\int d^2\boldsymbol{q}_T W(q_T) = \int d^2\boldsymbol{q}_T \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}} \widetilde{W}(b_*)$$

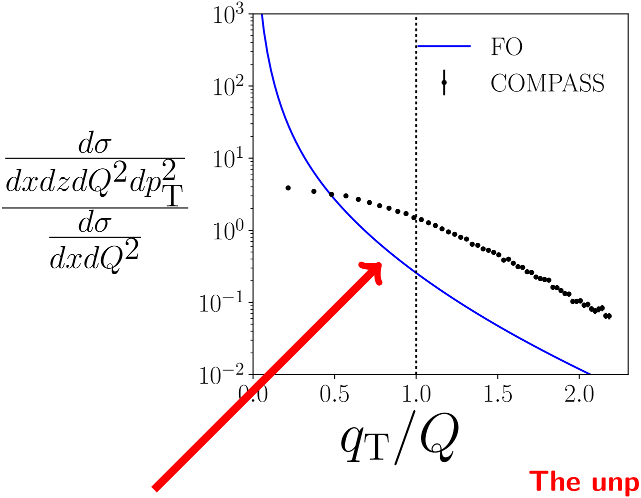
$$= \int d^2\boldsymbol{b}\; \delta^{(2)}(\boldsymbol{b})\; \widetilde{W}(b_*)$$

$$= 0$$

solution:

$$b_c(b) = \sqrt{b^2 + b_0^2/(C_5 Q)^2} \qquad \widetilde{W}(b_*) \to \widetilde{W}(b_c(b_*))$$

Does it work?



- Kinematics

  $Q^2 = 1.92 \text{ GeV}^2$

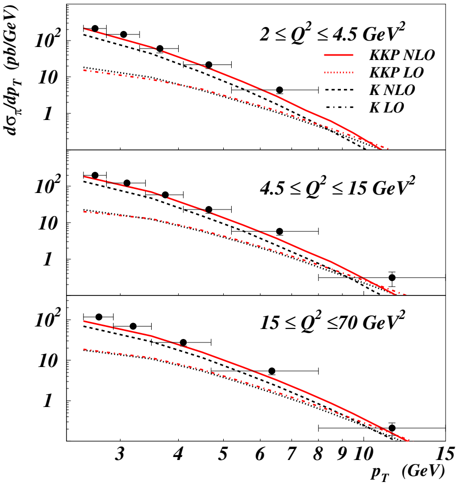
  $x = 0.0318$

  $z = 0.375$

- Need order α_S^2 or beyond?
- Soft gluon resummation?
- Subleading power corrections?

The unpolarized SIDIS cross sections needs to be ready to interpret upcoming TMD data from JLab 12

order α_S^2 corrections to FO

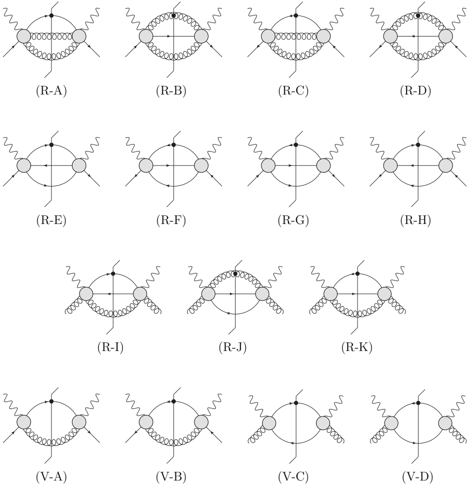


- There are strong indications that order α_S^2 corrections are very important
- The seems to give an order of magnitude of corrections at small p_T
- It is possible that the Y term improves significantly
- Unfortunately, codes are not publicly available

Daleo,et al. (2005)
PRD.71.034013

order α_S^2 corrections to FO

Gonzalez, Rogers, NS, Wang (2018)

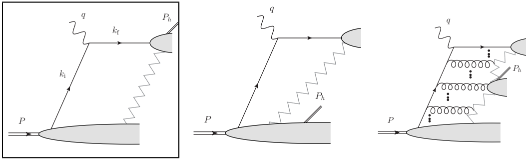


- We are calculating order α_S^2 corrections to the Y term ourselves
- Preliminary calculation of all the contributions are available which shows cancellation of all IR singularities
- At present we are checking carefully the results

Analysis of SIDIS kinematics

Boglione, Collins, Gamberg, Gonzalez, Rogers, NS (2017) PLB 766

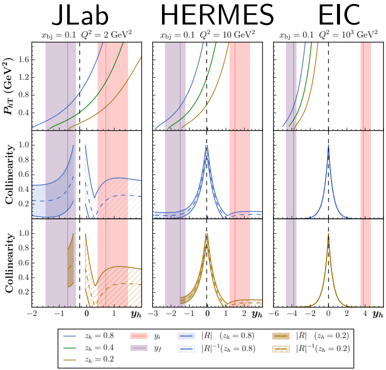
- **Can we apply factorization theorems in SIDIS measurements?**
- Factorization demands that



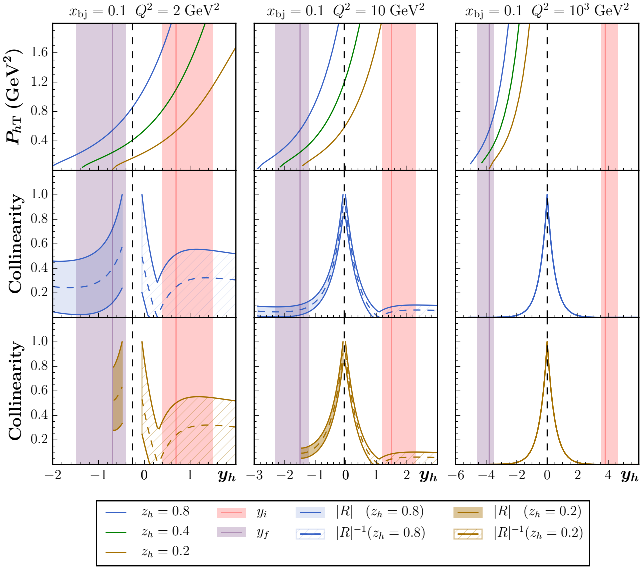
$$p_h \cdot k_f = \mathcal{O}(m^2)$$
$$p_h \cdot k_i = \mathcal{O}(Q^2)$$

- Define a *collinearity* parameter

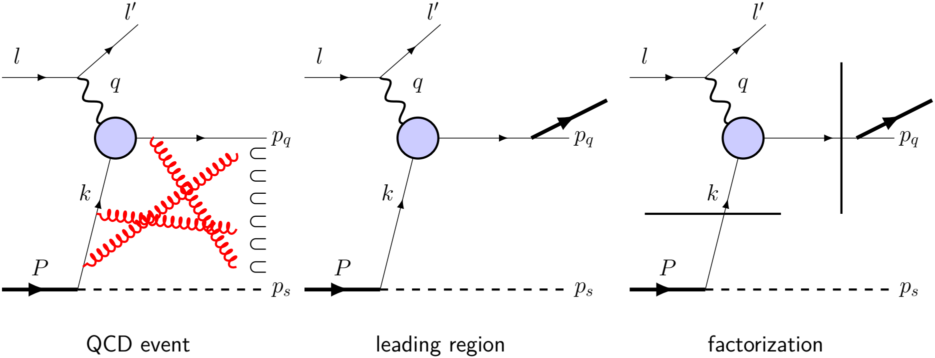
$$R = \frac{(p_h \cdot k_f)}{(p_h \cdot k_i)} = \mathcal{O}(m^2/Q^2)$$



Analysis of SIDIS kinematics

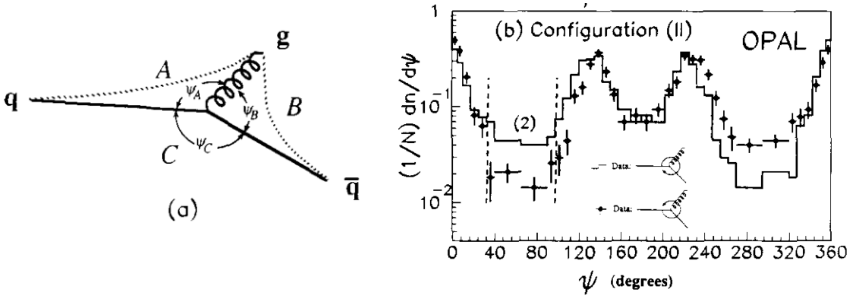


Beyond W+Y



- Factorization theorem (current fragmentation): $d\sigma/d\phi = H \otimes f \otimes D$
- Hadrons can also be produced in the mid rapidity region $\rightarrow$ see discussion by J. Collins arXiv:1610.09994
- String type effects are potentially important

String effects: PLB261 (1991) (OPAL Collaboration)



- 3 Jets events: $Q\bar{Q}$ and gluon jets. Jets are projected into a plane
- ψ: angle of a given particle relative to the quark jet with the highest energy
- ψ_A: angle between highest energetic jet and gluon jet
- ψ_C: angle between quark jets
- Only events with $\psi_A = \psi_C$ are kept
- **Particle flow asymmetry is observed $\rightarrow$ evidence of string effects**

Summary and outlook

$$\frac{d\sigma}{dx\, dy\, d\Psi\, dz\, d\phi_h\, dP_{hT}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\sum_{i=1}^{18} F_i(x,z,Q^2,P_{hT}^2)\beta_i$$

F_i	Standard label	β_i
F_1	$F_{UU,T}$	1
F_2	$F_{UU,L}$	ε
F_3	F_{LL}	$S_{\|\|}\lambda_e\sqrt{1-\varepsilon^2}$
F_4	$F_{UT}^{\sin(\phi_h+\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h+\phi_S)$
F_5	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sin(\phi_h-\phi_S)$
F_6	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h-\phi_S)$
F_7	$F_{UU}^{\cos 2\phi_h}$	$\varepsilon\ \cos(2\phi_h)$
F_8	$F_{UT}^{\sin(3\phi_h-\psi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(3\phi_h-\phi_S)$
F_9	$F_{LT}^{\cos(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)$
F_{10}	$F_{UL}^{\sin\ 2\phi_h}$	$S_{\|\|}\varepsilon\sin(2\phi_h)$
F_{11}	$F_{LT}^{\cos\phi_S}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S$
F_{12}	$F_{LL}^{\cos\phi_h}$	$S_{\|\|}\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h$
F_{13}	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)$
F_{14}	$F_{UL}^{\sin\phi_h}$	$S_{\|\|}\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h$
F_{15}	$F_{LU}^{\sin\phi_h}$	$\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h$
F_{16}	$F_{UU}^{\cos\phi_h}$	$\sqrt{2\varepsilon(1+\varepsilon)}\ \cos\phi_h$
F_{17}	$F_{UT}^{\sin\phi_S}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S$
F_{18}	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)$

- I discussed the current status of F_{UU}
- At present, there is no successful description of data using W+Y
- There are many results where the success is shown by restricting the q_T range and avoiding the inclusion of Y
- To make use of all the data from the upcoming JLab 12, we need to make more progress in theory to describe all the 18 structure functions