Lattice Calculations of 3D Structure

David Richards
Jefferson Laboratory

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INT Program “Spatial and Momentum Tomography of Hadrons and Nuclei”

Introduction

- Measures of Hadron Structure and Lattice QCD
- 1-D hadron Structure - Parton Distribution Functions and Form Factors
- 3-D Measures: (Moments of)
  - Generalized Parton Distributions
  - TMDs
- New Developments in LQCD: LaMET, Quasi-distributions, Pseudo-Distributions
- Summary

Lattice QCD

Observables in lattice QCD are then expressed in terms of the path integral as

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \prod_{n,\mu} dU_\mu(n) \prod_n d\psi(n) \prod_n d\bar{\psi}(n) \mathcal{O}(U, \psi, \bar{\psi}) e^{-(S_G[U] + S_F[U,\psi,\bar{\psi}])}$$

Integrate out the Grassmann variables:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \prod_{n,\mu} dU_\mu(n) \mathcal{O}(U, G[U]) \det M[U] e^{-S_G[U]}$$

Importance Sampling

where $G(U, x, y)^{ij}_{\alpha\beta} \equiv \langle \psi^i_\alpha(x) \bar{\psi}^j_\beta(y) \rangle = M^{-1}(U)$

- Generate an ensemble of gauge configurations

$$P[U] \propto \det M[U] e^{-S_G[U]}$$

This is REAL for Euclidean space QCD - *but see later*

- Calculate observable

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U^n, G[U^n])$$

Measures of Hadron Structure

5D

Wigner distributions

Transverse Momentum Dependent Distributions (TMDs)

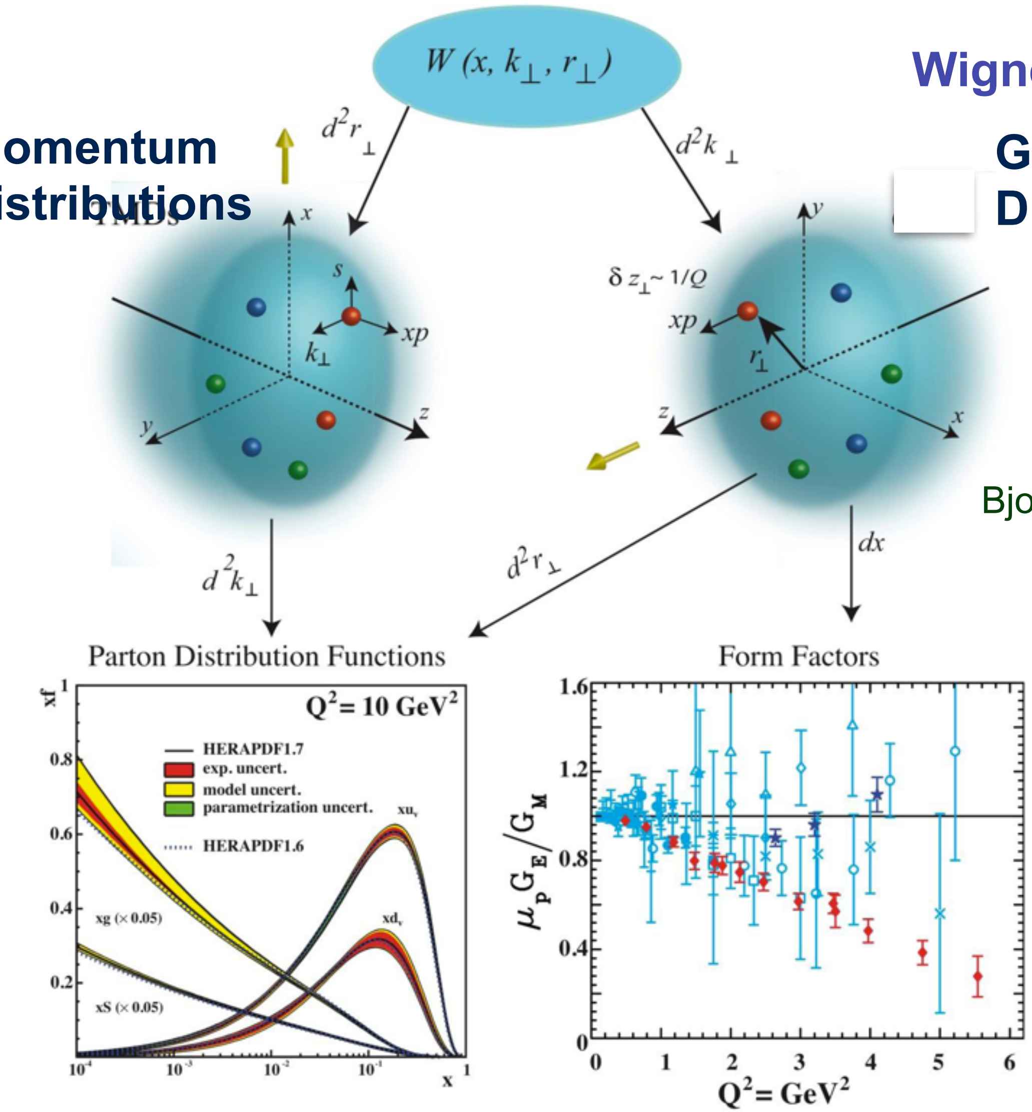
Generalized Parton Distributions (GPDs)

3D

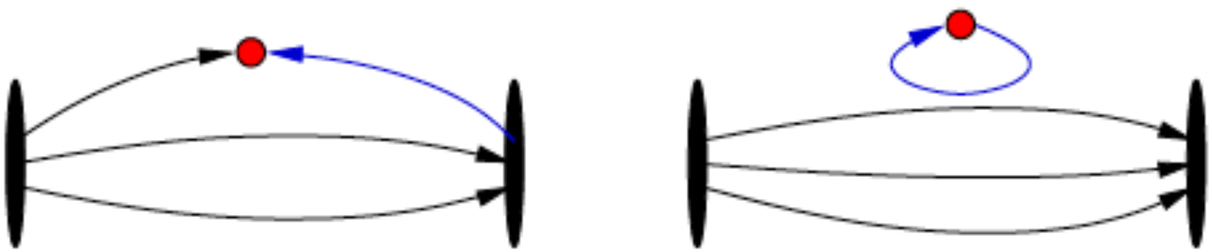
Bjorken-x and transverse momentum

Bjorken-x and impact parameter

1D



Hadron Structure



$$C_{3\mathrm{pt}}(t_{sep}, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0 \mid N(\vec{x}, t_{\mathrm{sep}}) V_\mu(\vec{y}, t) \bar{N}(\vec{0}, 0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

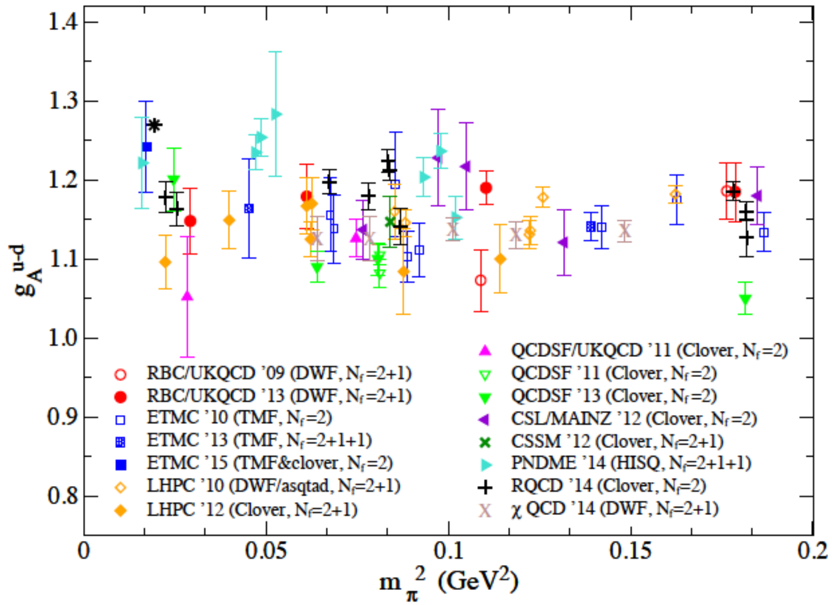
Resolution of unity – insert states

$$\longrightarrow \langle 0 \mid N \mid N, \vec{p}+\vec{q} \rangle \langle N, \vec{p}+\vec{q} \mid V_\mu \mid N \vec{p} \rangle \langle N, \vec{p} \mid \bar{N} \mid 0 \rangle e^{-E(\vec{p}+\vec{q})(t_{\mathrm{sep}}-t)} e^{-E(\vec{p})t}$$

One-Dimensional Structure

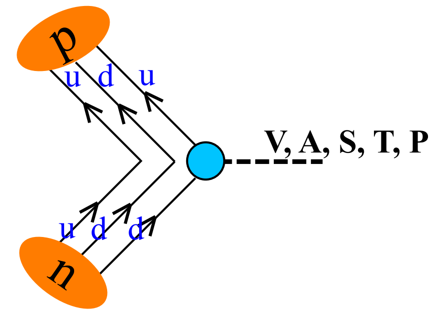
1D Structure - Charges

M Constantinou, arXiv:1511.00214



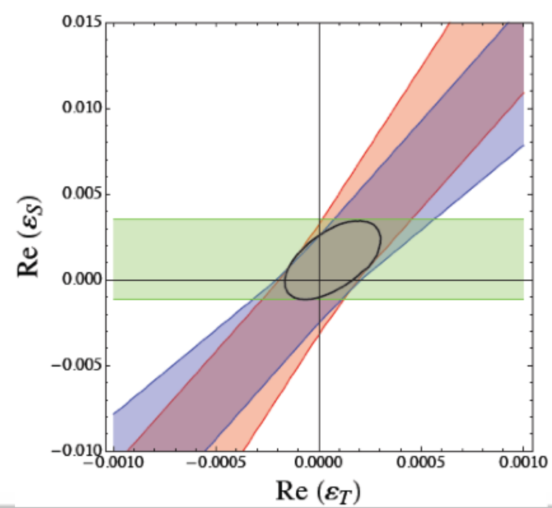
- Governs beta-decay rate
- Important for proton-proton fusion rate in solar models
- Benchmark for lattice QCD calculations of hadron structure

e.g. novel interactions probed in ultra-cold neutron decay



$$H_{eff} \supset G_F\left[\varepsilon_S\,\bar{u}d \times \bar{e}(1-\gamma_5)\nu_e + \varepsilon_T\,\bar{u}\sigma_{\mu\nu}d \times \bar{e}\sigma^{\mu\nu}(1-\gamma_5)\nu_e\right]$$

$$g_S = Z_S \langle p|\bar{u}d|n\rangle \quad g_T = Z_T \langle p|\bar{u}\sigma_{\mu\nu}d|n\rangle$$



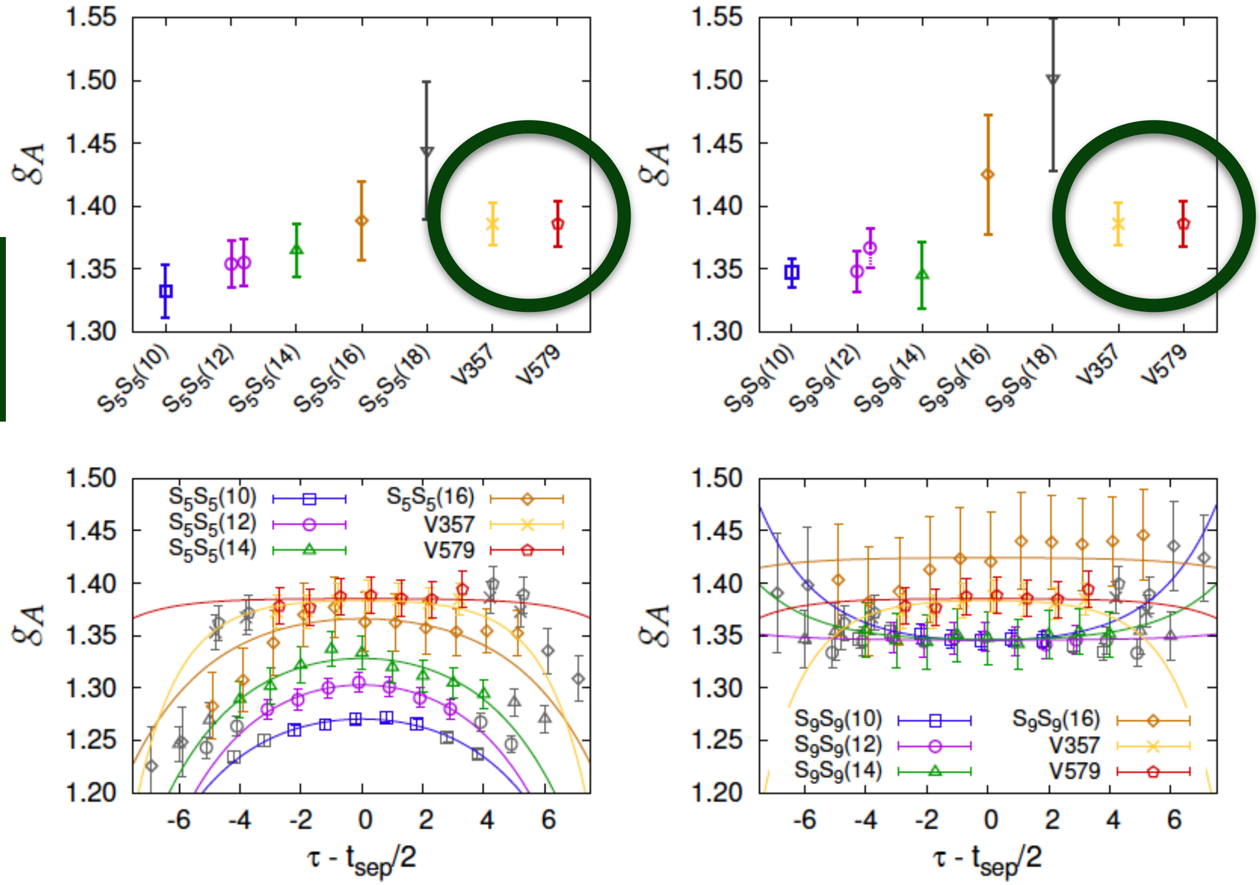
R Gupta, 2014

Systematic Uncertainties

Yoon et al., Phys. Rev. D 93, 114506 (2016)

Failure to isolating **ground state** leads to important systematic uncertainty.

Variational Method



Renormalized Charges

Yoon et al., Phys. Rev. D 95, 074508 (2017)

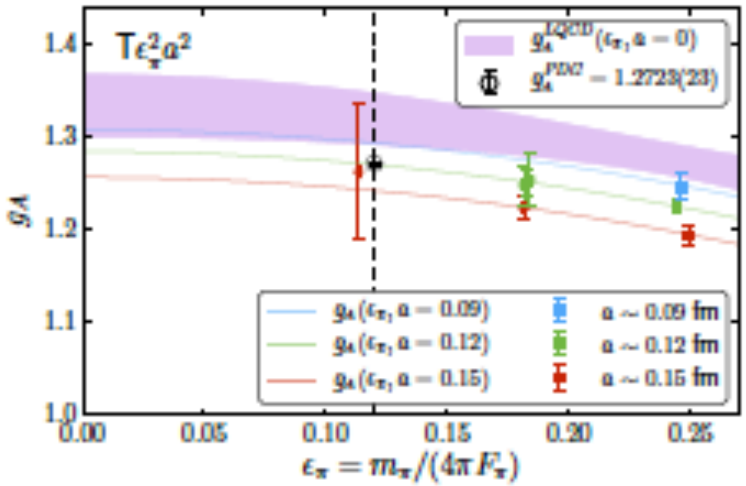
ID	Lattice Theory	a fm	M_π(MeV)	g_A^{u-d}	g_S^{u-d}	g_T^{u-d}	g_V^{u-d}
$a127m285$	2+1 clover-on-clover	0.127(2)	285(6)	1.249(28)	0.89(5)	1.023(21)	1.014(2
$a12m310$	2+1+1 clover-on-HISQ	0.121(1)	310(3)	1.229(14)	0.84(4)	1.055(36)	0.969(2
$a094m280$	2+1 clover-on-clover	0.094(1)	278(3)	1.208(33)	0.99(9)	0.973(36)	0.998(2
$a09m310$	2+1+1 clover-on-HISQ	0.089(1)	313(3)	1.231(33)	0.84(10)	1.024(42)	0.975(3
$a091m170$	2+1 clover-on-clover	0.091(1)	166(2)	1.210(19)	0.86(9)	0.996(23)	1.012(2
$a09m220$	2+1+1 clover-on-HISQ	0.087(1)	226(2)	1.249(35)	0.80(12)	1.039(36)	0.969(3
$a09m130$	2+1+1 clover-on-HISQ	0.087(1)	138(1)	1.230(29)	0.90(11)	0.975(38)	0.971(3

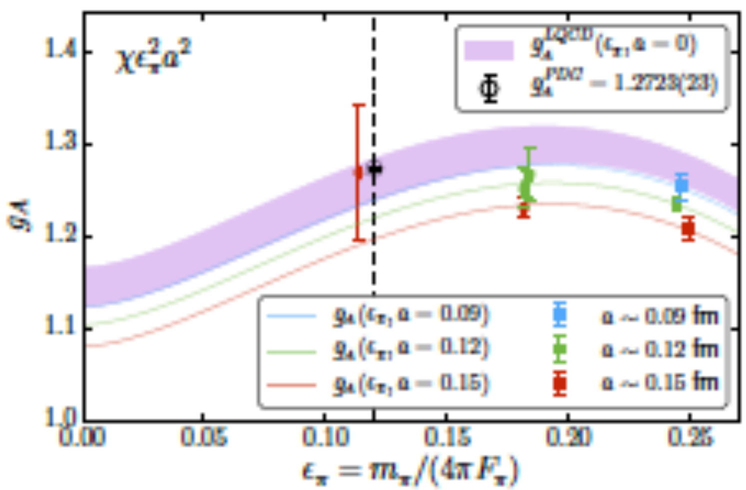
Consistency between different actions

Matrix Elements of 1st excited state?

ID	Type	$\langle 0\|\mathcal{O}_A\|1\rangle$	$\langle 0\|\mathcal{O}_S\|1\rangle$	$\langle 0\|\mathcal{O}_T\|1\rangle$	$\langle 0\|\mathcal{O}_V\|1\rangle$	$\langle 1\|\mathcal{O}_A\|1\rangle$	$\langle 1\|\mathcal{O}_S\|1\rangle$	$\langle 1\|\mathcal{O}_T\|1\rangle$	$\langle 1\|\mathcal{O}_V\|1\rangle$
		−0.179(21)		0.182(16)		−0.9(2.4)		−0.2(1.2)	
			−0.35(4)		−0.014(2)		0.6(1.1)		0.80(34)
a127m285	S_5S_5	−0.172(18)	−0.37(4)	0.210(15)	−0.015(2)	0.75(48)	0.8(9)	0.42(27)	0.87(28)
		−0.295(58)	−0.45(15)	0.167(40)	−0.014(6)	1.5(3.0)	1.8(1.4)	0.54(86)	0.86(55)
		−0.295(57)	−0.45(15)	0.166(47)	−0.014(6)	1.46(54)	1.8(1.4)	0.54(41)	0.86(28)

Feynman-Hellman Method





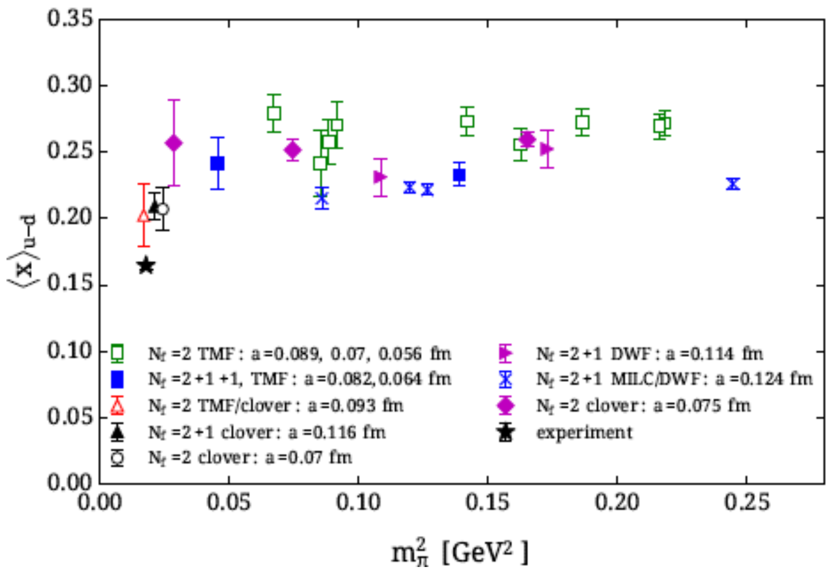
Berkowitz et al, arXiv:1704.01114

Calculation using Feynman-Hellman Theory

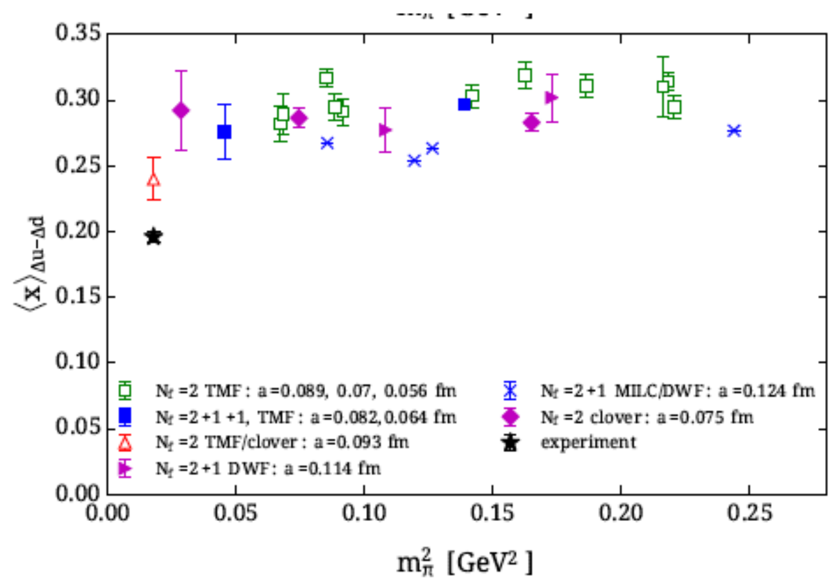
$$H = H_0 + \lambda H_\lambda$$
$$\frac{\partial E_n}{\partial \lambda} = \langle n \mid H_\lambda \mid n \rangle$$

Reduces to calculation of energy-shift of two-point functions ***but*** repeat the calculation for each operator

Isovector Moments of PDFs

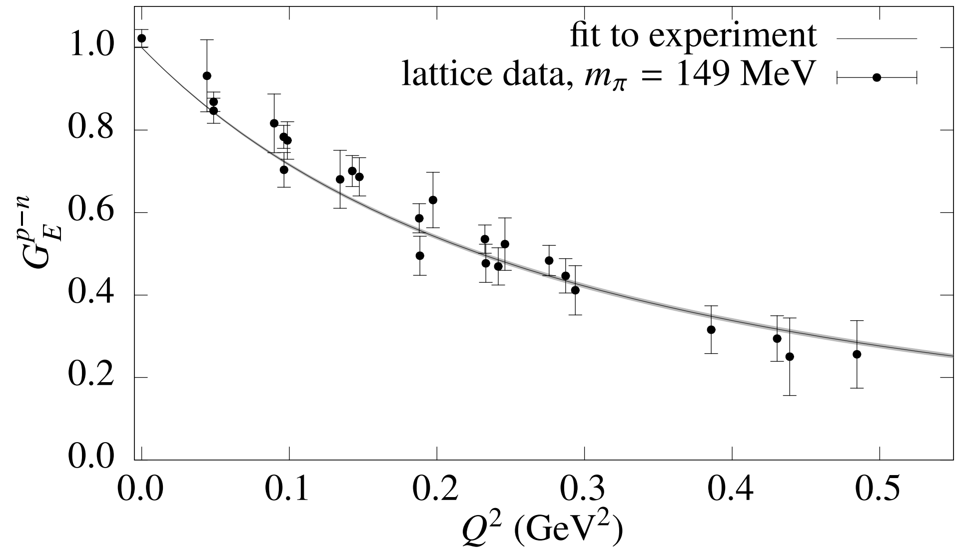


Abdel-Rehim et al, Phys. Rev. D 93, 039904 (2016)



1D Structure: EM Form Factors

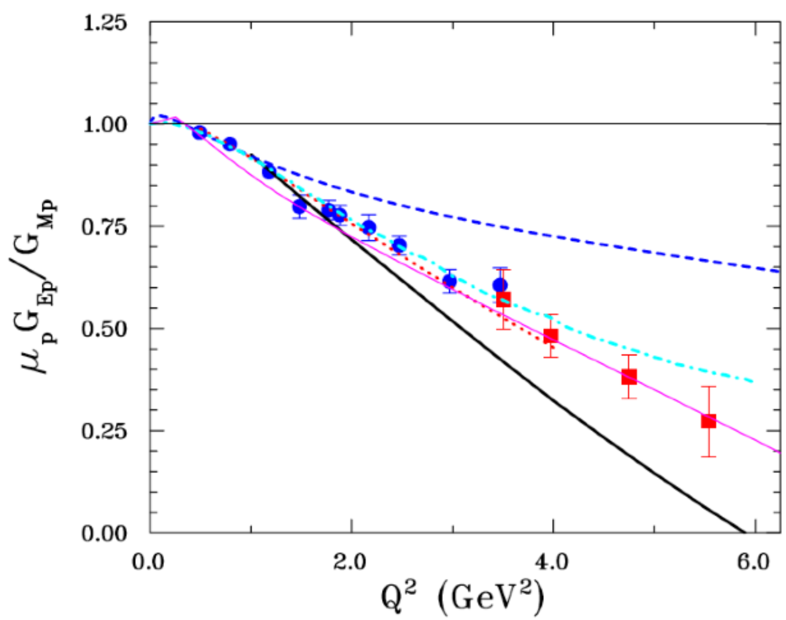
Wilson-clover lattices from BMW



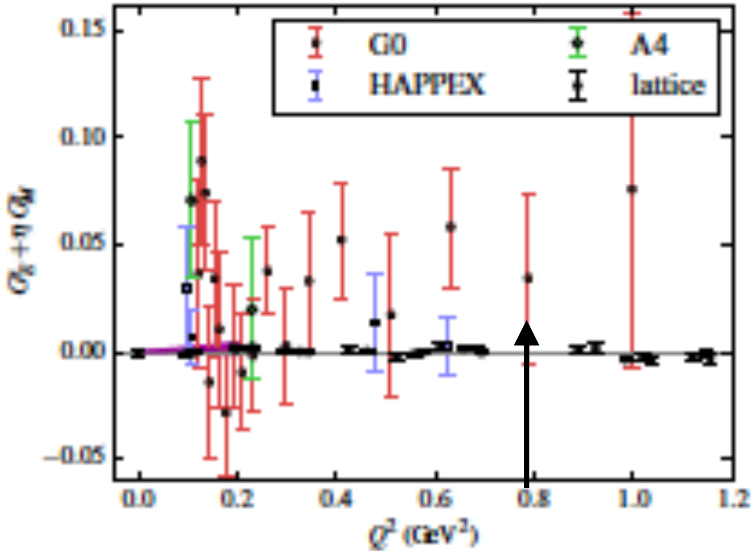
Green et al (LHPC), Phys. Rev. D 90, 074507 (2014)

Hadron structure at nearly-physical quark masses

Large Q^2 behavior: Hall C at JLab to 15 GeV^2



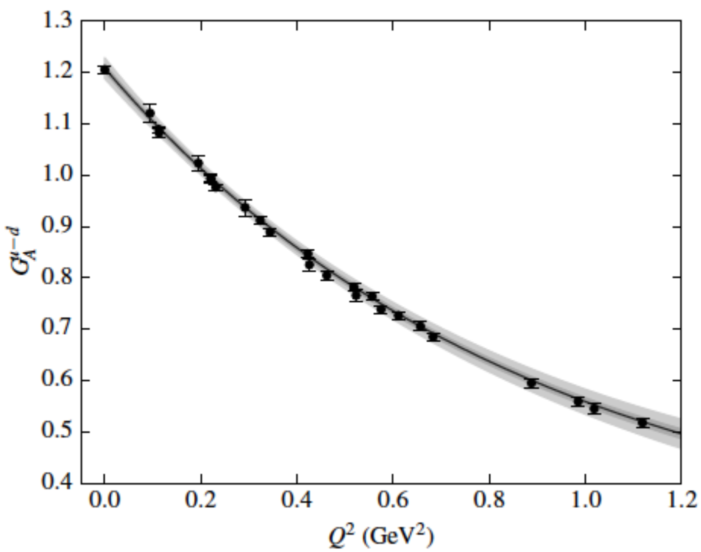
Sea Quark Contributions

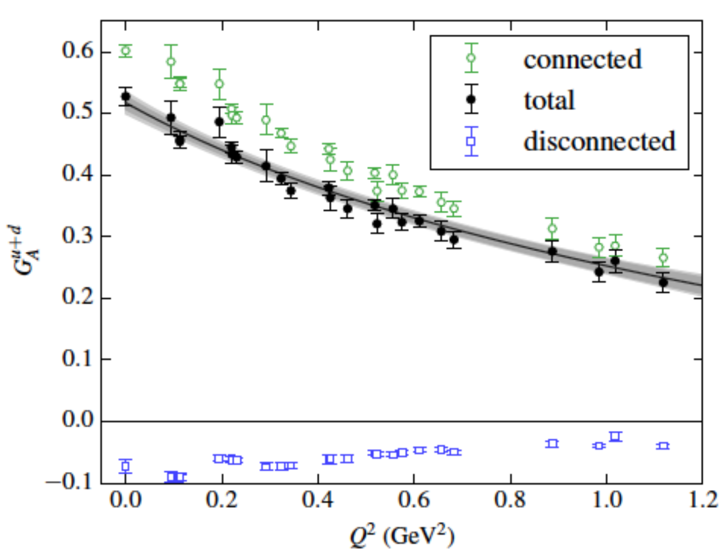


J. Green, K. Orginos *et al.*, Phys. Rev. D 92, 031501 (2015); Phys. Rev. D 95, 114502 (2017

Using *Hierarchical Probing* - A. Stathopoulos, J. Laeuchli, K. Orginos (2013)

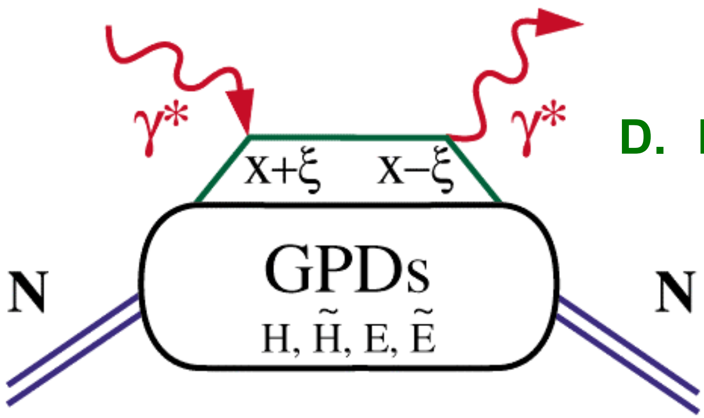
Combination *measured* in expt





Generalized Parton Distributions

- Measured in *Deeply-Virtual Compton Scattering* (DVCS) and Exclusive Meson Production.



D. Muller *et al* (1994), X. Ji, Radyushkin (1996)

$$\bar{u}(P')\left(\gamma^+ H(x,\xi,t) + i\frac{\sigma^{+k}\Delta_k}{2m}E(x,\xi,t)\right)u(P) =$$
$$\int_{-\infty}^{\infty}\frac{d\omega^-}{4\pi}e^{-i\xi P^+\omega^-}\langle P' \mid T\bar{\psi}(0,\omega^-,O_{\mathrm{T}})W(\omega^-,0)\gamma^+\frac{\lambda^a}{2}\psi(0)\mid P\rangle$$

GPDs - II

- Light-cone distributions not accessible in Euclidean-space QCD

$$\int_{-1}^{1} dx\, x^{n-1} \left[\begin{array}{c} H(x,\xi,t) \\ E(x,\xi,t) \end{array} \right] = \sum_{k=0}^{(n-1)/2} (2\xi)^{2k} \left[\begin{array}{c} A_{n,2k}(t) \\ B_{n,2k}(t) \end{array} \right] \pm \delta_{n,\mathrm{even}} (2\xi)^n C_n(t)$$

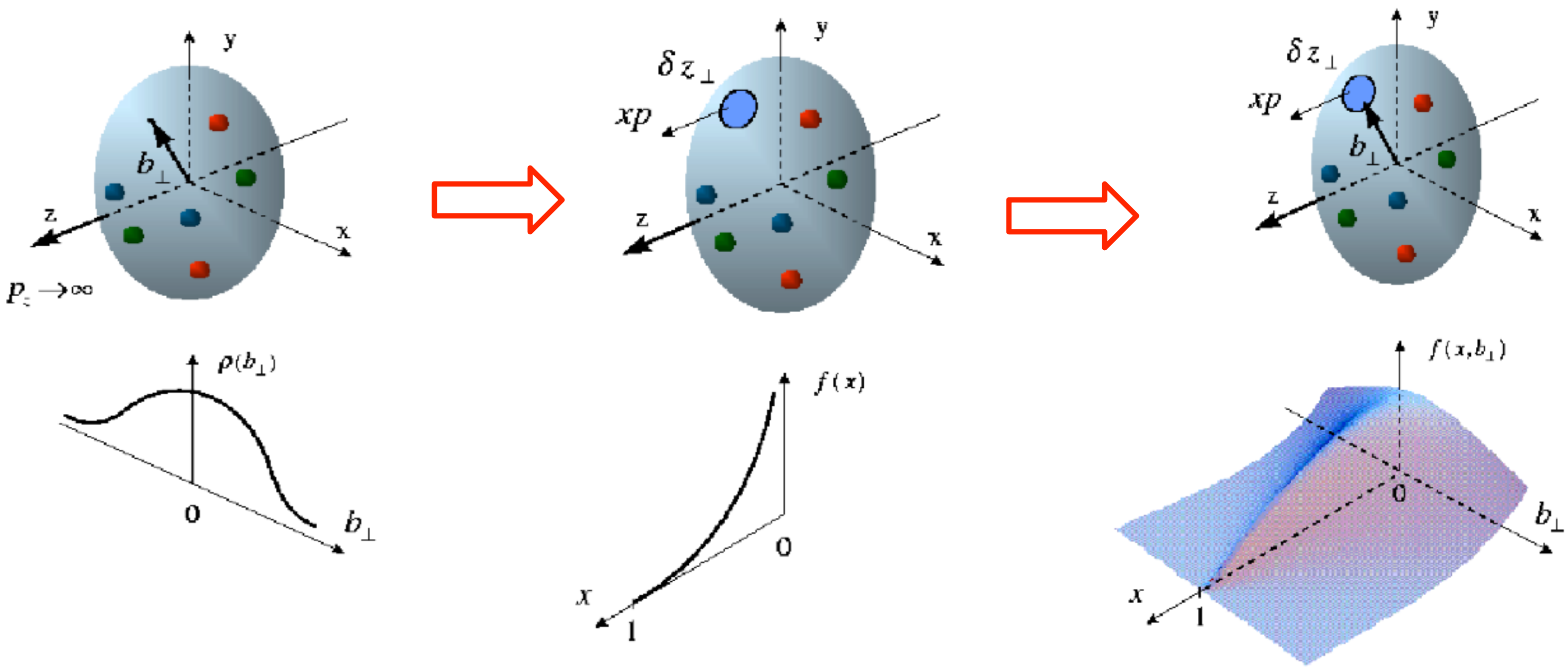
Generalized *Form Factors*

- Related to matrix elements of *local operators*

$$\mathcal{O}^{\mu_1\ldots\mu_n} = i^{n-1} \bar{\psi} \gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n\}} \frac{\lambda^a}{2} \psi$$

Higher Moments restricted by hypercubic symmetry

Different Regimes in Different Experiments

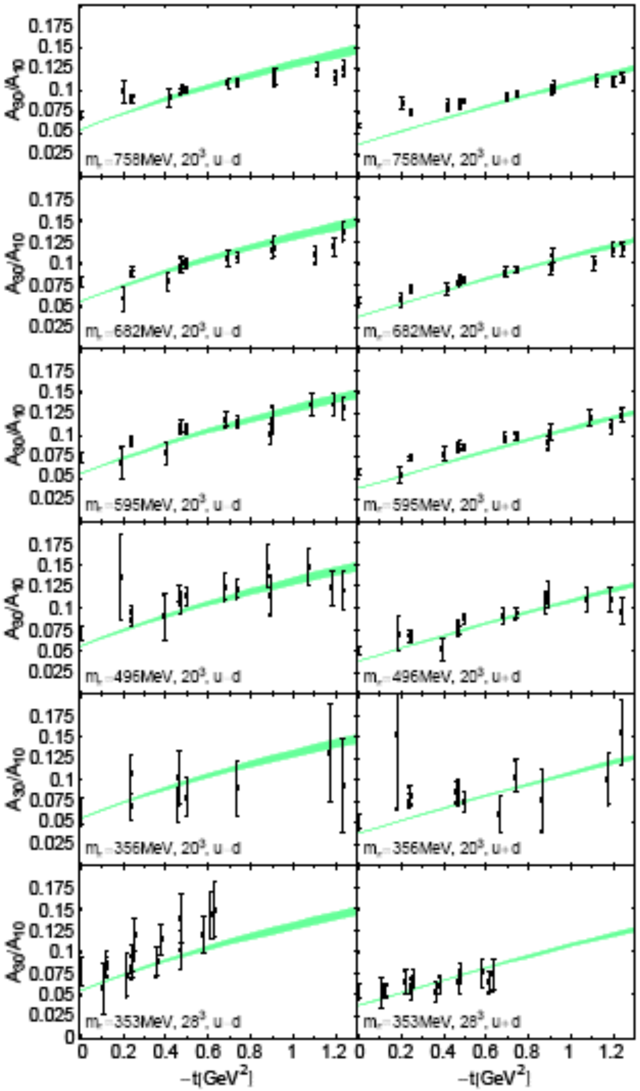


Form Factors
transverse quark distribution in Coordinate space

Structure Functions
longitudinal quark distribution in momentum space

GPDs
Fully-correlated quark distribution in both coordinate and momentum space

Parametrizations of GPDs



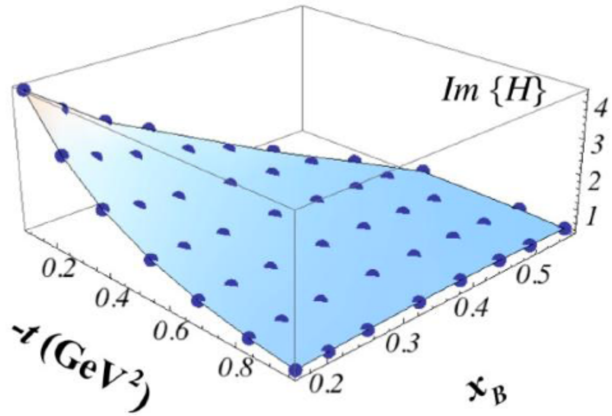
LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); Phys.Rev.D82:094502,2010

Provide phenomenological guidance for GPD's

– ***CTEQ, Nucleon Form Factors, Regge***

Comparison with *Diehl et al, hep-ph/0408173*

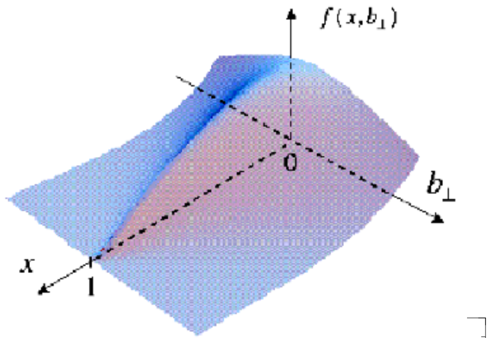
Important Role for LQCD

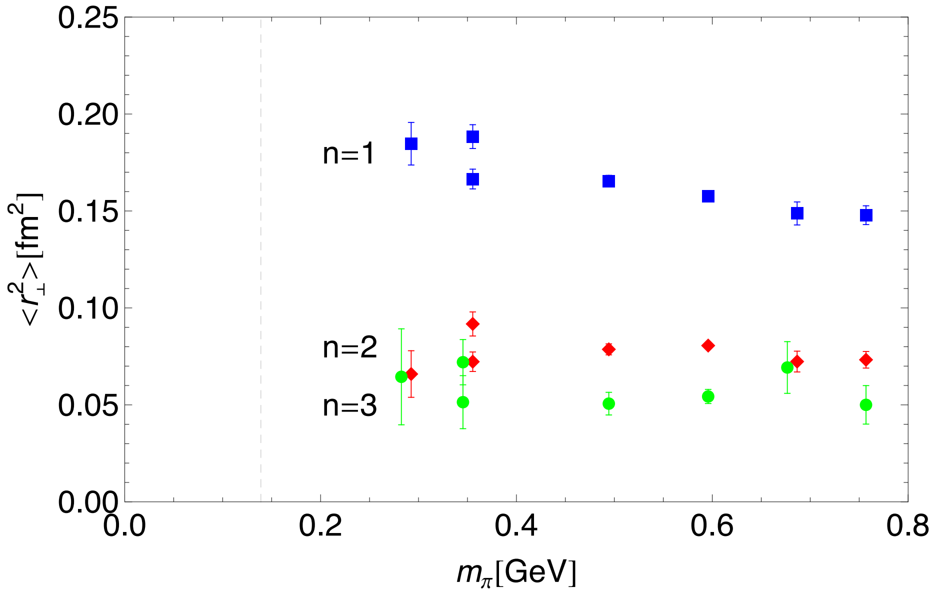


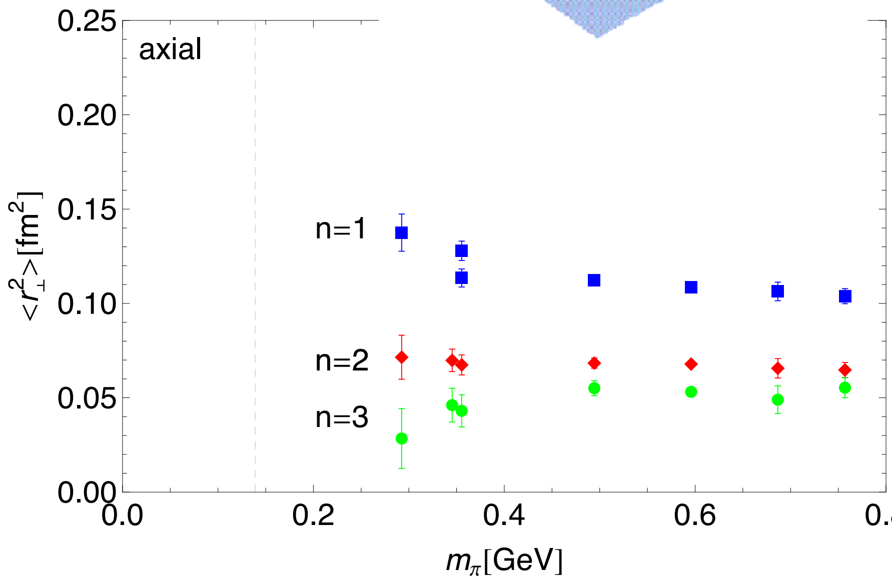
Charge Radius of GFFs

Lattice results consistent with narrowing of transverse size with increasing x

Flattening of GFFs with increasing n







GPDs and Orbital Angular Momentum

- **Form factors of energy momentum tensor - *quark and gluon angular momentum***

$$``\bar{q}\gamma_\mu D_\nu q"$$

$$\frac{1}{2} = \sum_q J^q + J^g$$

X.D. Ji, PRL 78, 610 (1997)

$$= \frac{1}{2}\left\{\sum_q \left(A^q_{20}(t=0) + B^q_{20}(t=0)\right) + A^g_{20}(t=0) + B^g_{20}(t=0)\right\}$$

$$\sum_q \left(\frac{1}{2}\Delta\Sigma^q + L^q\right)$$

Decomposition

- **Gauge-invariant**
- **Renormalization-scale dependent**
- **Handle on Quark orbital angular momentum**

Mathur et al., *Phys.Rev. D62 (2000) 114504*

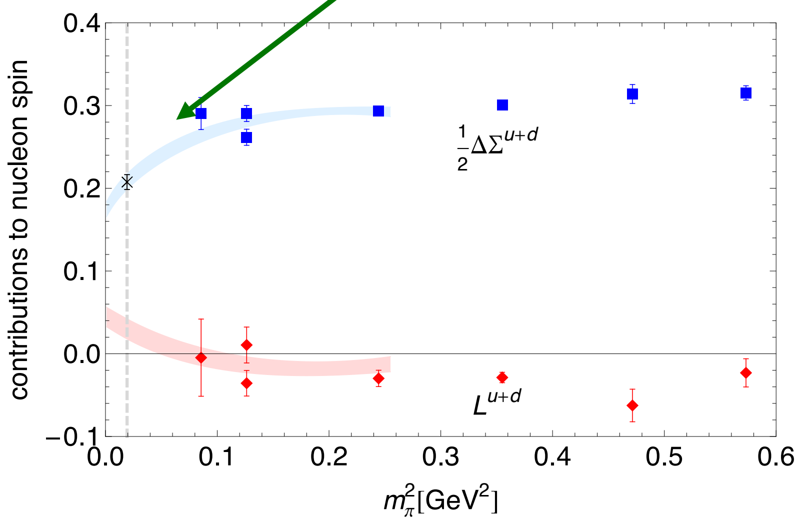
Origin of Nucleon Spin

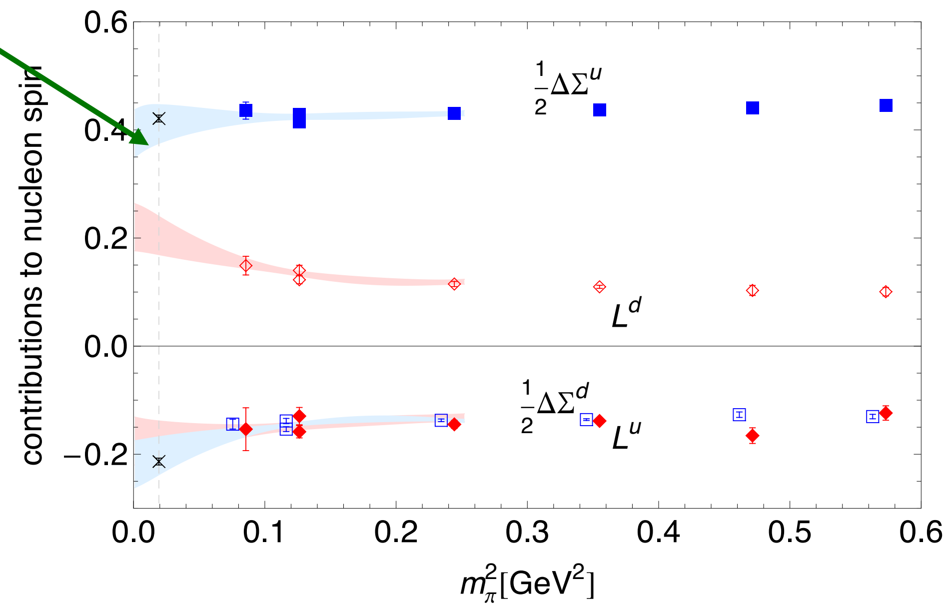
- Total orbital angular momentum carried by quarks small
- Orbital angular momentum carried by individual quark flavours substantial.

$$
\begin{aligned}
J^q &= 1/2\left(A^q_{20}(t=0) + B^q_{20}(t-0)\right) \\
\Delta\Sigma^q/2 &= \tilde{A}^q_{10}(t=0)/2 \\
\frac{1}{2} &= \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g
\end{aligned}
$$

LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); arXiv.1001.3620

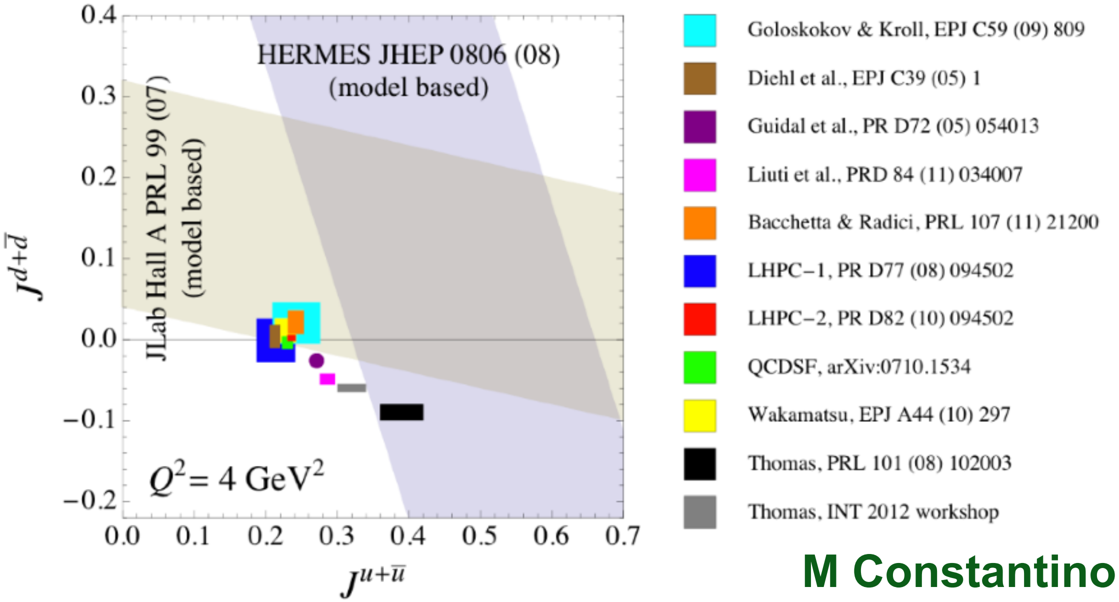
HERMES, PRD75 (2007)



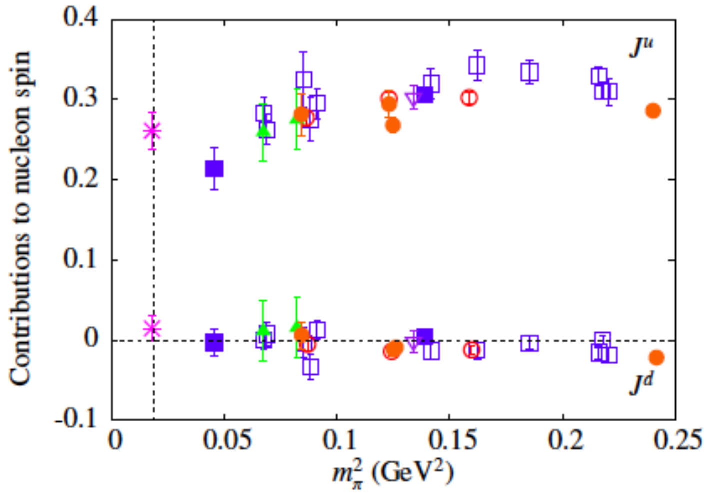


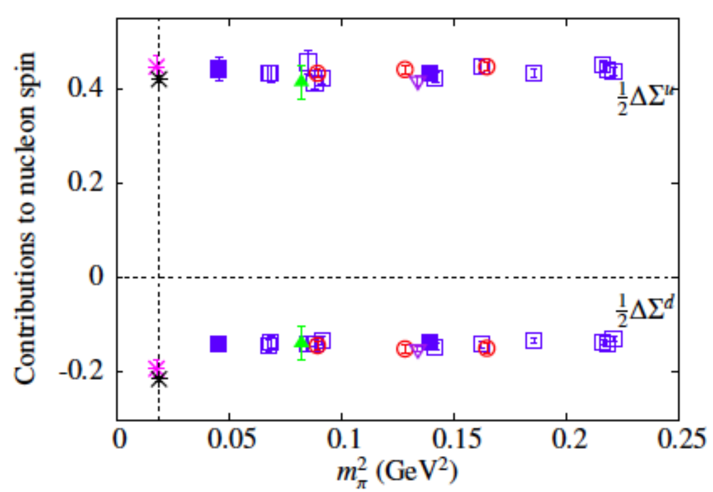
Disconnected contributions neglected.

Origin of Nucleon Spin - II



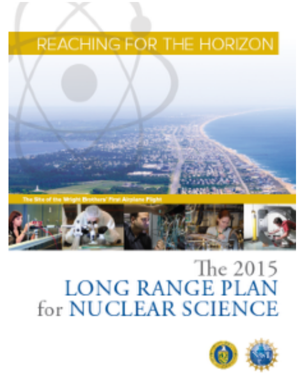
M Constantinou, arXiv:1511.00214

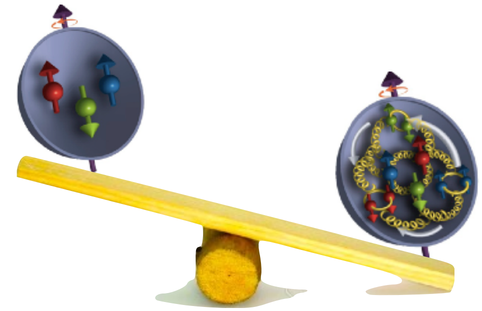




Energy-Momentum Tensor

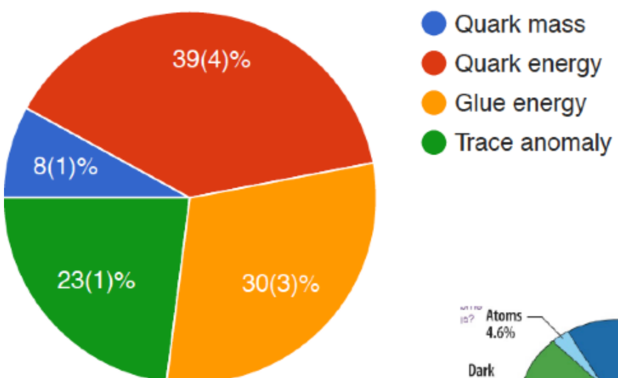
“Understanding the Glue That Binds Us All: The Next QCD Frontier in Nuclear Physics”



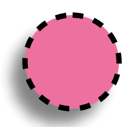


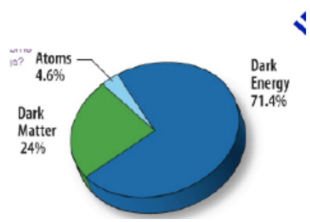
- Quark masses contribute only 1% to mass of proton: binding through gluon confinement
- Gluon spin and orbital angular momentum to spin of proton largely unknown

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}D_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2; \langle P \mid T_{\mu\nu} \mid P\rangle = P_\mu P_\nu / M$$



$$\text{Trace Anomaly: } T_{\mu\mu} = -(1+\gamma_m)\bar{\psi}\psi + \frac{\beta(g)}{2g}G^2$$

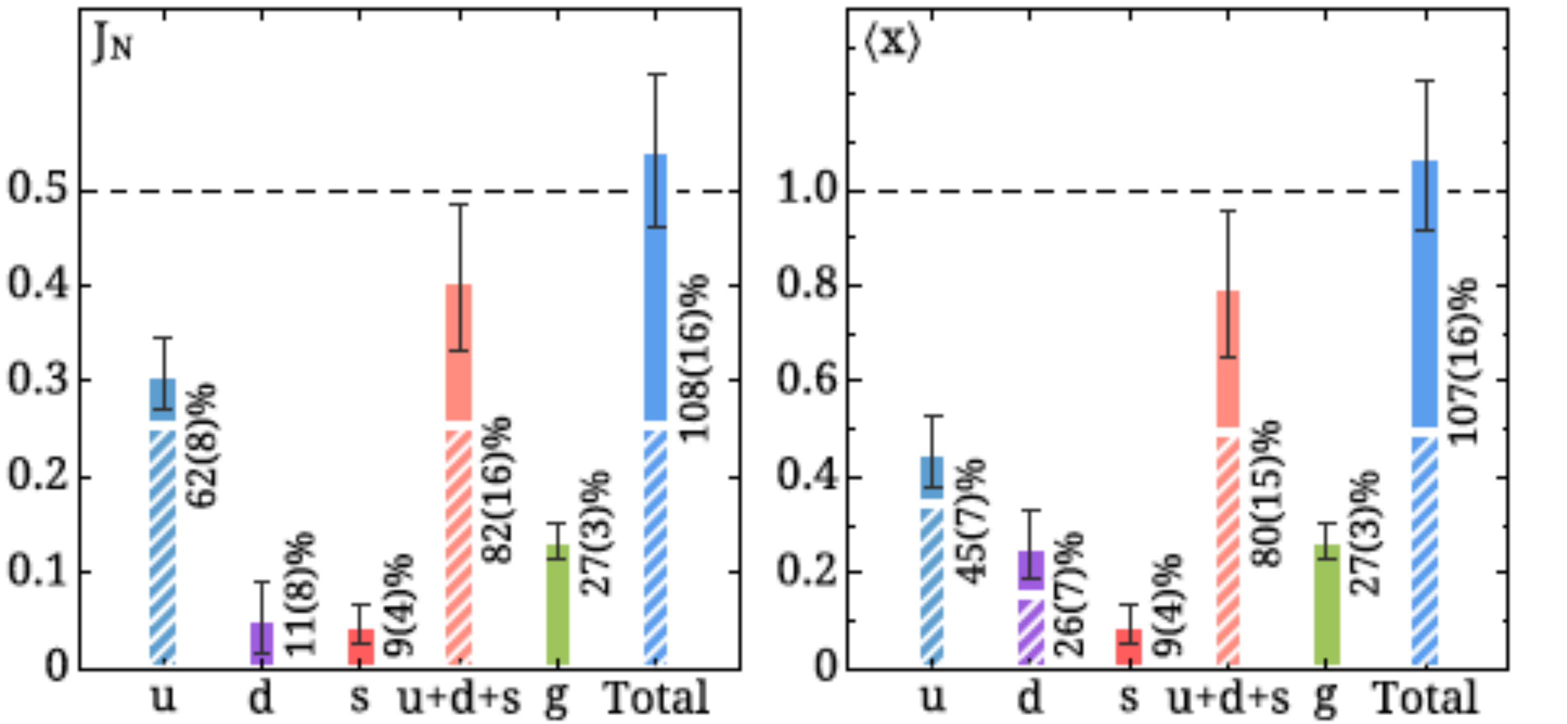




Yang, Trento 2017

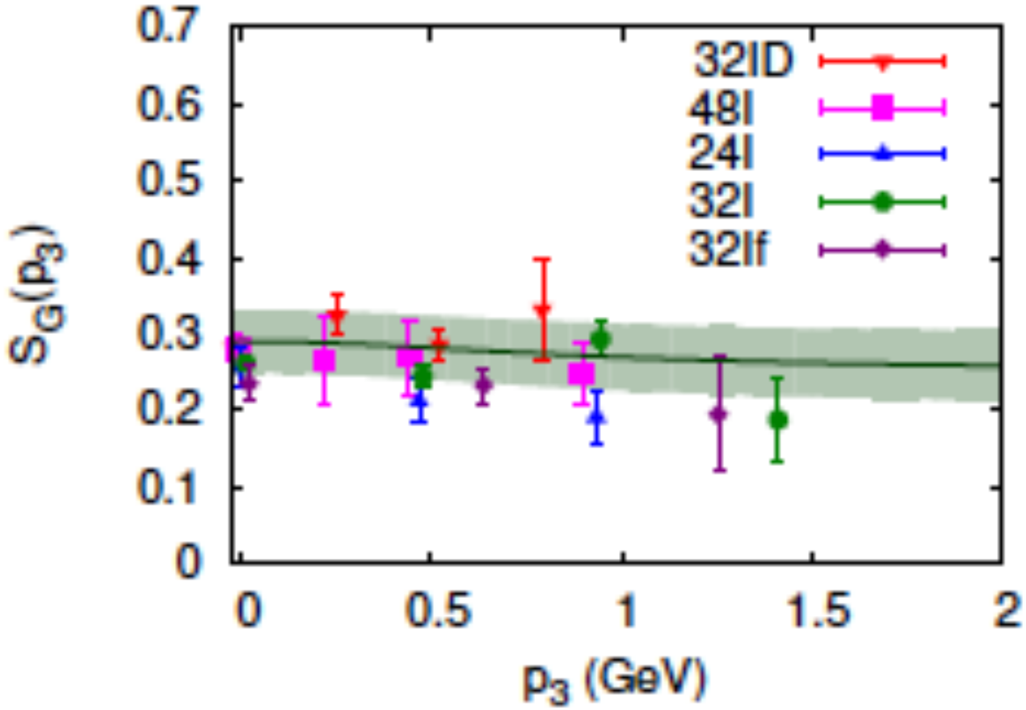
Spin and Momentum Decomposition

Twisted-Mass Fermions: C.Alexandrou et al, arXiv:1706.02973



→ Momentum and Spin Sum Rules Satisfied

Gluon Spin



Yang et al, Phys. Rev. Lett. 118, 102001 (2017)

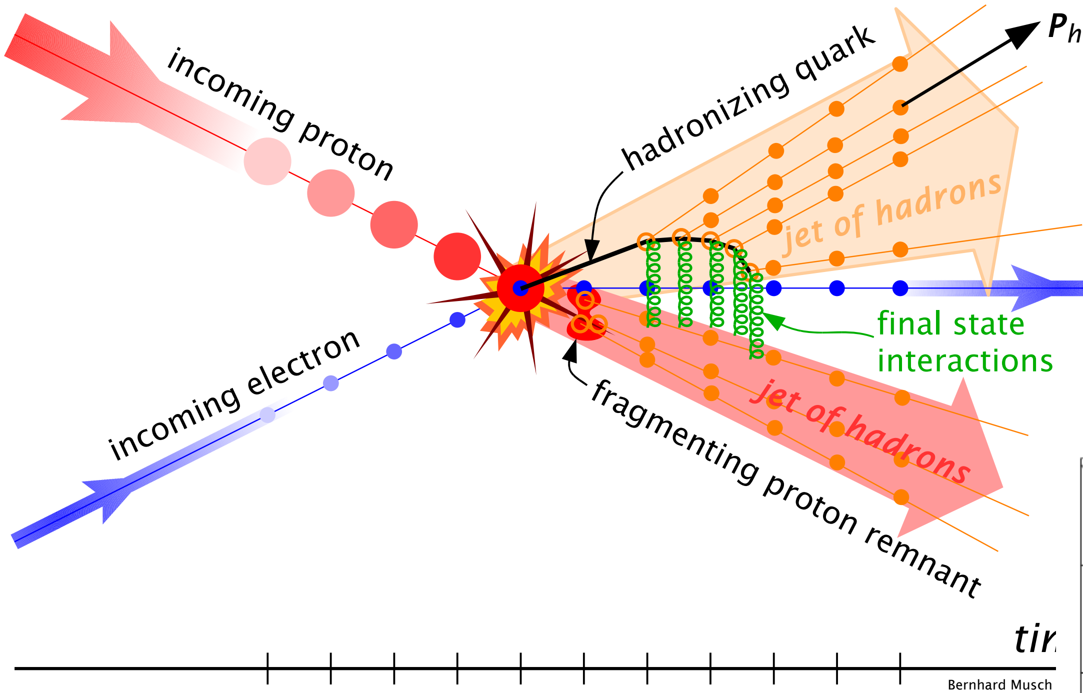
$$\vec{S}_g = 2\int d^3x\,\mathrm{Tr}(\vec{E}_c \times \vec{A}_c)$$

ΔG in large p limit

Transverse momentum distributions (TMDs)

from experiment, e.g., **SIDIS** (semi-inclusive deep inelastic scattering) + DY

HERMES, COMPASS, JLab 12 GeV , RHIC-spin, EIC, DY



Slide: B. Musch

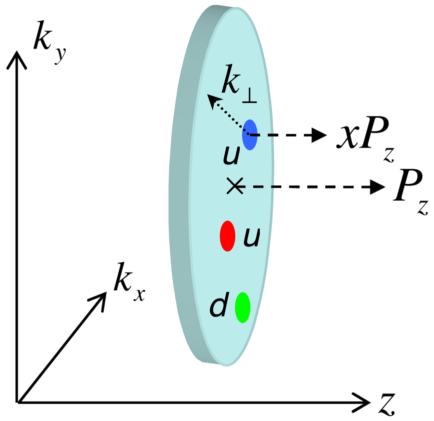
$N \backslash q$	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Boer-Mulders

Sivers ← time-reversal odd

final state interactions!
explain large asymmetries otherwise forbidden!
signature of QCD!

TMDs in Lattice QCD



B. Musch, PhD Thesis; Haegler, Musch, Negele, Schafer arXiv:0908.1283

Introduce Momentum-space correlators

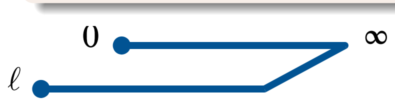
$$
\begin{aligned}
\Phi_\Gamma &= \int d(n\cdot k)\int \frac{d^4 l}{2(2\pi)^4} e^{-ik\cdot l}\tilde{\Phi}_\Gamma(l;P,S) \\
&= \int d(n\cdot k)\int \frac{d^4 l}{2(2\pi)^4} e^{-ik\cdot l}\langle P,S\,|\,\bar{q}(l)\Gamma\mathcal{U}q(0)\,|\,P,S\rangle
\end{aligned}
$$

continuum

$$\mathcal{U} \equiv \mathcal{P}\exp\left(-ig\int_0^\ell d\xi^\mu A_\mu(\xi)\right)$$

along path from 0 to ℓ

Choice of path - retain gauge invariance



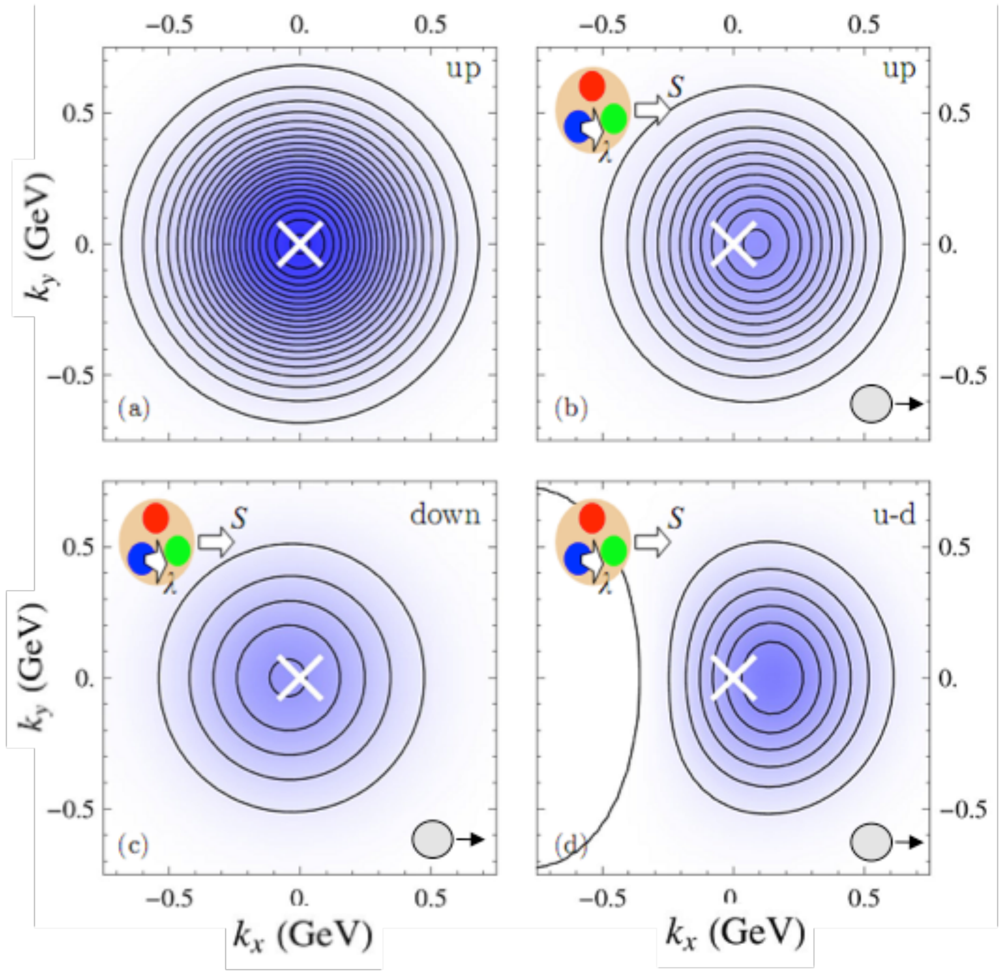
SIDIS: path runs to infinity

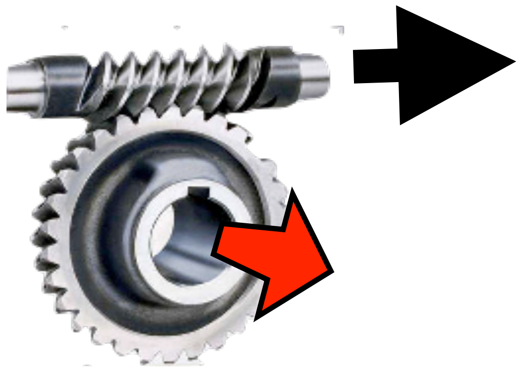
Lattice: equal time slice

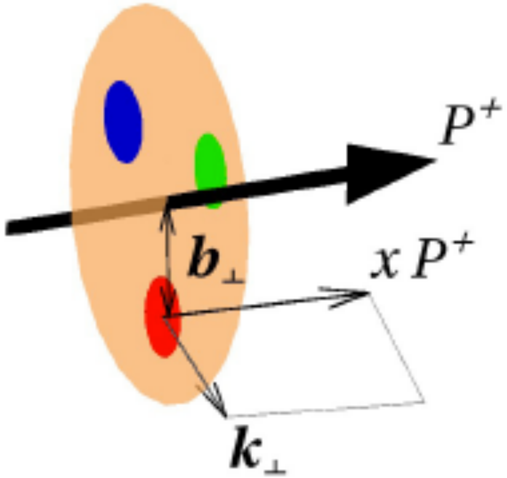
Slide: A. Bacchetta

Worm gears on the lattice

P. Hägler, B. U. Musch, J. W. Negele, and A. Schäfer, Europhys. Lett. 88 (2009) 61001

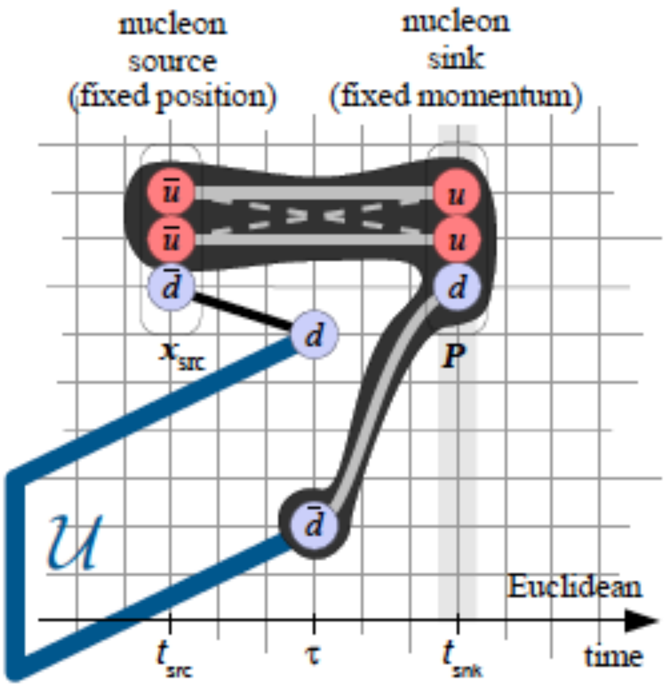


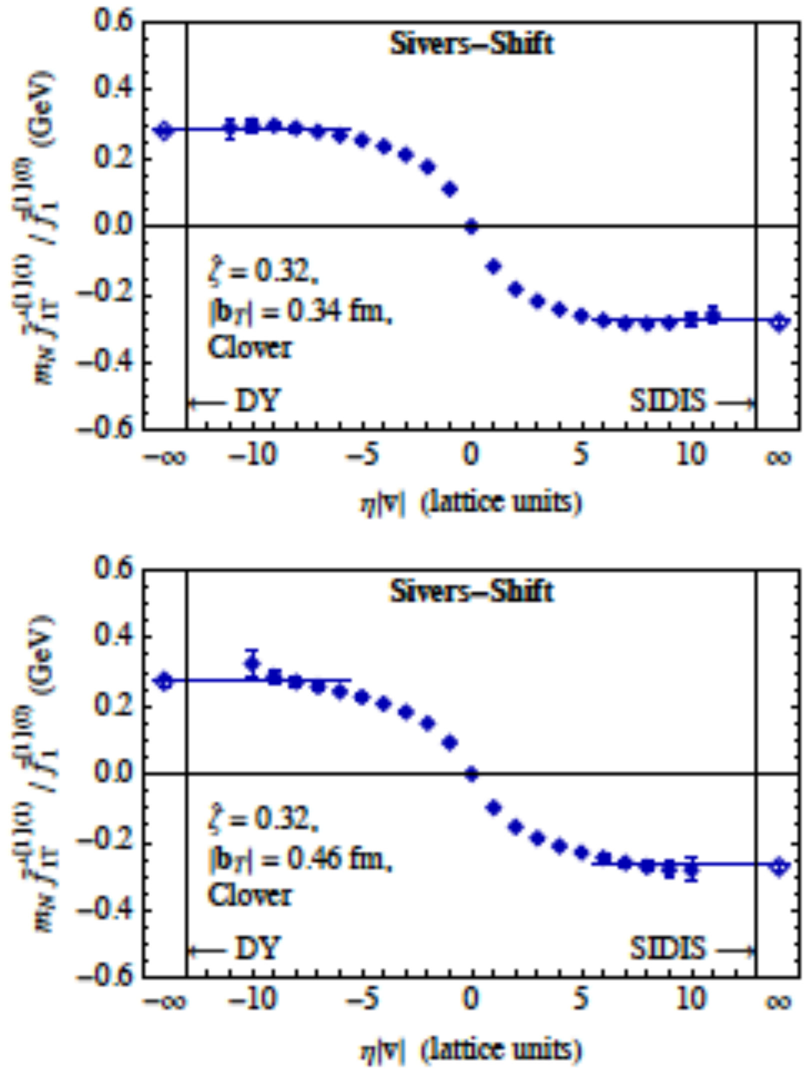




Transverse momentum distributions (TMDs)

Lattice QCD





B. Musch et al., Phys.Rev. D85 (2012) 094510;
M. Engelhardt, Lattice 2014
Yoon et al, arXiv:1706.03606

Direct Calculation of Bjorken-x Dependence

Two Challenges….

- Euclidean lattice precludes the calculation of light-cone correlation functions
  - So… …Use *Operator-Product-Expansion* to formulate in terms of *Mellin Moments* with respect to *Bjorken x*.

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) \mid P \rangle$$

$$\downarrow$$

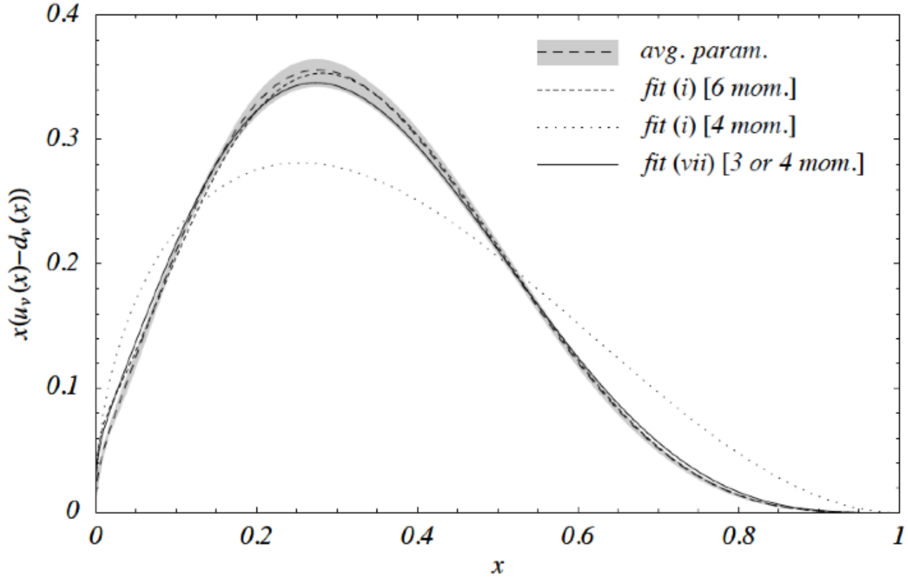
$$\langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5) D_{\mu_2} \dots D_{\mu_n} \psi \mid P \rangle \to P_{\mu_1} \dots P_{\mu_n} a^{(n)}$$

- *Generalized Parton Distributions (off-forward): GPDs*
- *Quark Distribution Amplitudes in exclusive processes: PDAs*
- *(Transverse-Momentum-Dependent Distributions): TMDs*

- Discretisation, and hence reduced symmetry of the lattice, introduces power-divergent mixing for N >3 moment.

Higher Moments of Parton Distributions

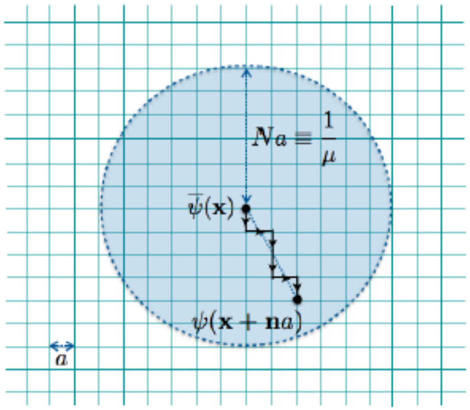
$$x(u_v(x) - d_v(x)) = ax^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma x)$$



IsoVector Distribution

Need to constrain parameters from phenomenology.

Detmold, Melnitchouk, Thomas
Eur.Phys.J.direct C3:1-15,2001



Use **improved, extended operators** to reduce power-divergent mixing. c.f. restoration of rotational symmetry for interpolating operators in spectroscopy

Davoudi and Savage, PRD86, 054505 (2012)

Quasi Distributions

- A solution, LaMET (Large Momentum Effective Theory) was proposed by X.Ji

X. Ji, Phys. Rev. Lett. 110 (2013) 262002

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')}\psi(0) \mid P > \\ + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

- Quasi distributions approach light-cone distributions in limit of large P^z

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

Y-Q Ma and J-W Qiu, arXiv:1404.6860

- Matching and evolution of quasi- and light-cone distributions

Carlson, Freid, arXiv:1702.05775
Isikawa et al., arXiv:1609.02018
Monahan and Orginos, arXiv:1612.01584
Orginos, Radyushkin , et al arXiv:1706.05373 (Pseudo Distributions)
Briceno, Hansen, Monahan, arXiv:1703.06072 (Euclidean Signature)

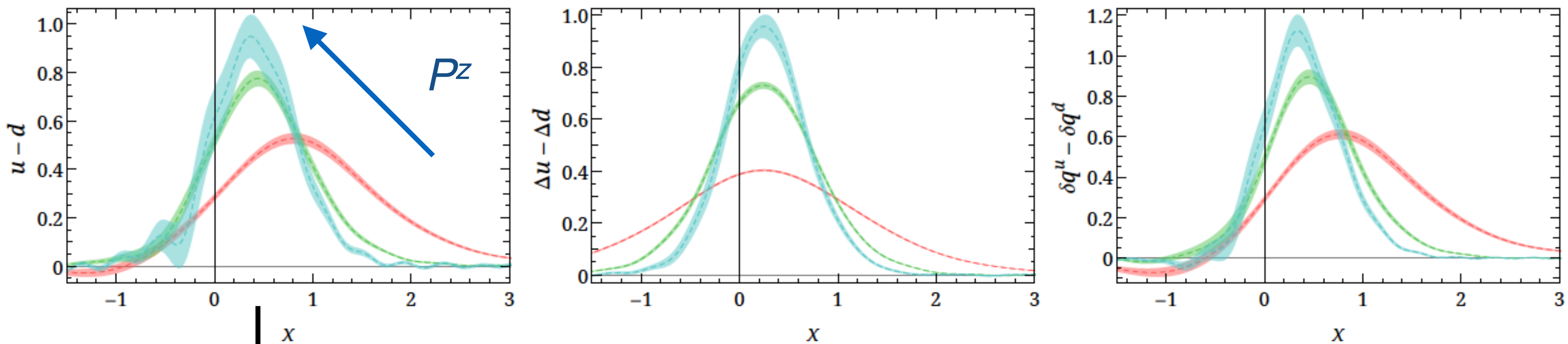
- Direct lattice calculation of hadronic tensor

K.F. Liu and S.J.Dong, PRL72, 1790 (1994); arXiv:1703.04690

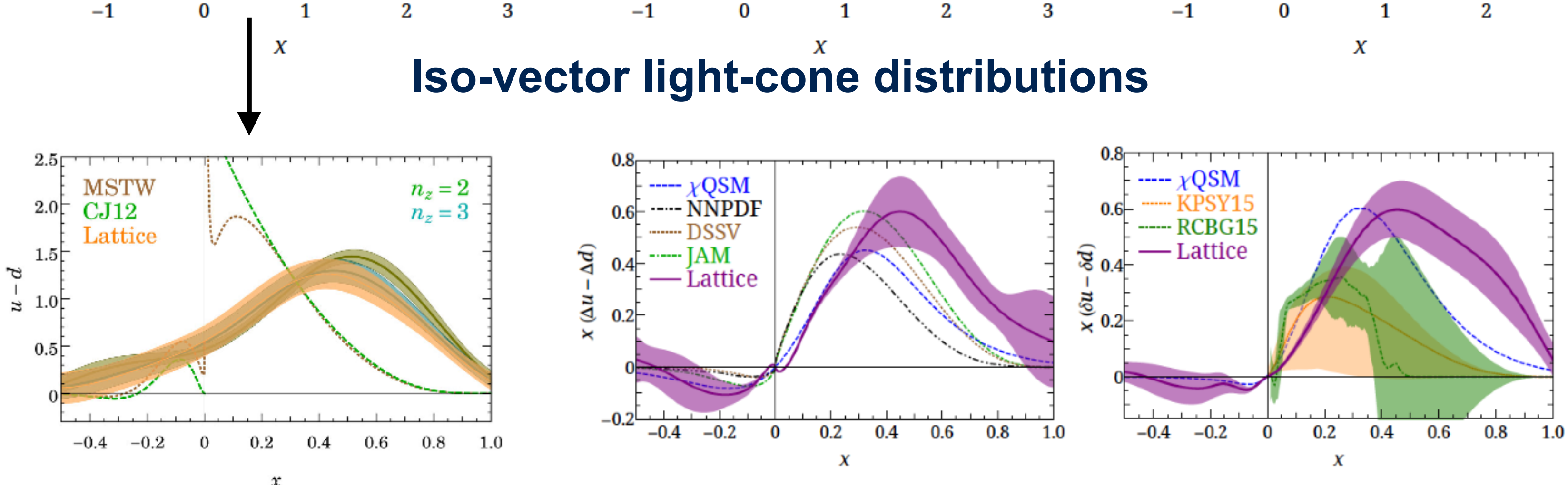
PDFs

H-W Lin, arXiv:1612.09366

Iso-vector quasi distributions



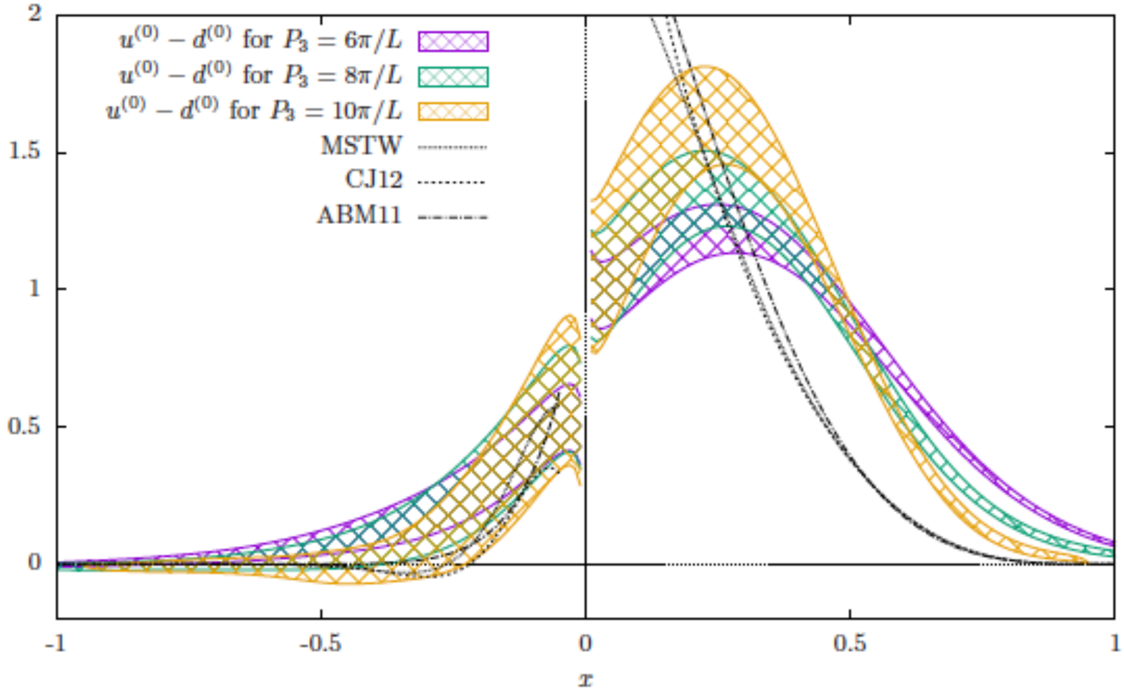
Iso-vector light-cone distributions



Yibo Yang, Friday

PDFs - II

Alexandrou et al., arXiv:1610.03689

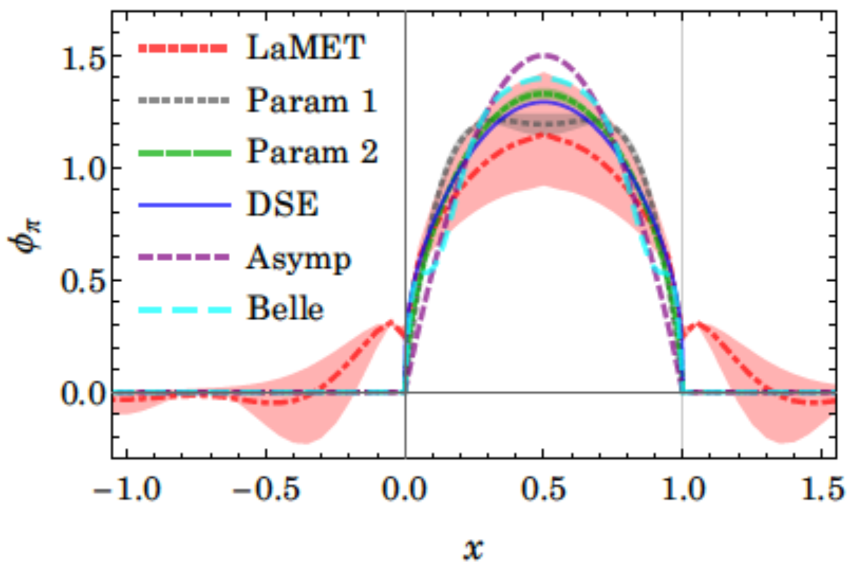


Unrenormalized PDFs
– Twisted-Mass Fermions
'High Statistics
Momentum-Smearing for
high momenta

Pion Distribution Amplitude

- Same operators as in polarized structure functions
- …BUT two-point function A. Radyushkin, Phys.Rev. D95 (2017) no.5, 056020
- Governs EM form factors at high Q^2

$$\phi_\pi(x) = \frac{i}{f_\pi} \int \frac{d\xi}{2\pi} e^{i(x-1)\xi\lambda\cdot P} \langle \pi(P) \mid \bar{\psi}(0)\lambda\cdot\gamma\gamma_5\Gamma(0,\xi\lambda\psi(\xi\lambda) \mid 0 >$$



Zhang et al., arXiv:1702.00008

SUMMARY

- Lattice Calculations now have controlled uncertainties for certain key benchmark quantities, and can confront experiment.
  - Ji's sum rule
  - TMDs
  - Narrowing of hadron with increasing x
- Near Frontiers
  - sea quark and *gluonic* contributions to hadron structure.
  - Direct calculations of Bjorken-x dependence
- Capitalizing on Expt + LQCD + Phenomenology