Matching TMD and collinear factorization

Andrea Signori

QCD structure
of the nucleon
in the modern era

UCLA
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Collaborators

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- Tomas Kasemets (Nikhef, VU Amsterdam)

- Jean-Philippe Lansberg (IPN Orsay, CNRS-IN2P3)

- Cristian Pisano (Pavia U.)

Outline of the talk

1) TMD & collinear PDFs

2) why do we need matching schemes ?

3) standard W+Y

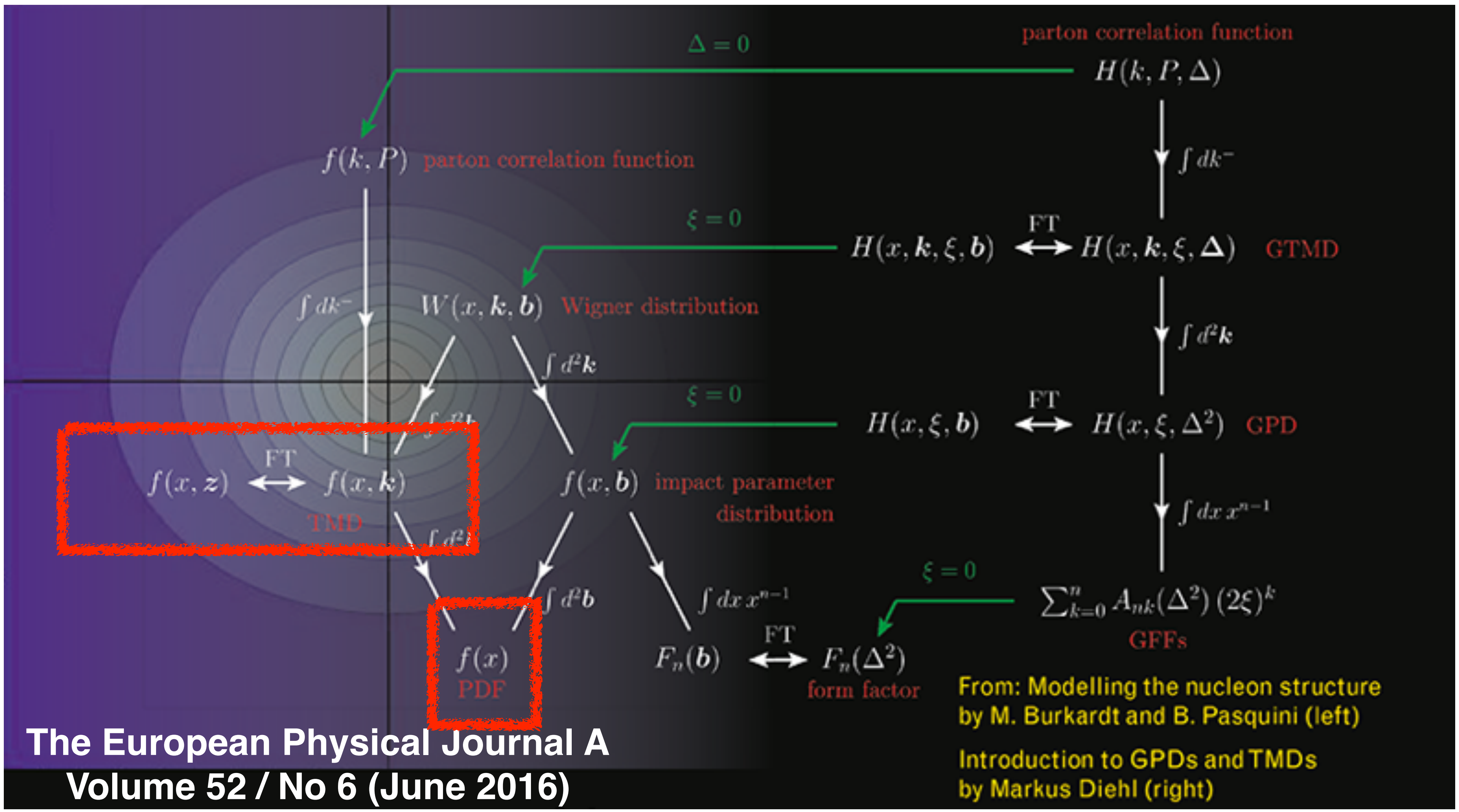
4) improved W+Y

5) log-average

6) discussion & feedback

Which hadron structure ?

Manifestation of hadron structure in scattering processes



Nature is “**smooth**” : understand the link between TMDs & PDFs

Why TMD & collinear distributions ?

Nucleon tomography in momentum space:
to understand how hadrons are built in terms of the elementary degrees of freedom of QCD

High-energy phenomenology:
to improve our understanding of high-energy scattering experiments and tackle BSM physics

A selection of open questions :

1) do we understand the interplay and the matching between collinear and TMD factorization ?

2) what's the impact of hadron structure on the determination of Standard Model parameters ?

3) how can we investigate gluon TMDs ?

4) what is the behavior of gluon TMDs at small-x ?

5) can we quantify factorization breaking effects ?

6) can we attempt a global fit of TMDs ?

7) can we access collinear and TMD distributions simultaneously ?

...

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Why TMD & collinear distributions ?

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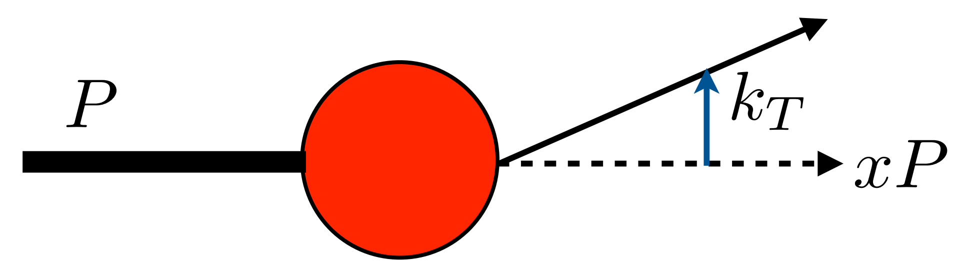
...

quark TMD PDFs

$$\Phi_{ij}(k, P; S) \sim \mathrm{F.T.}\ \langle PS \mid \bar{\psi}_j(0)\ U_{[0,\xi]}\ \psi_i(\xi)\ | PS \rangle_{|LF}$$

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$



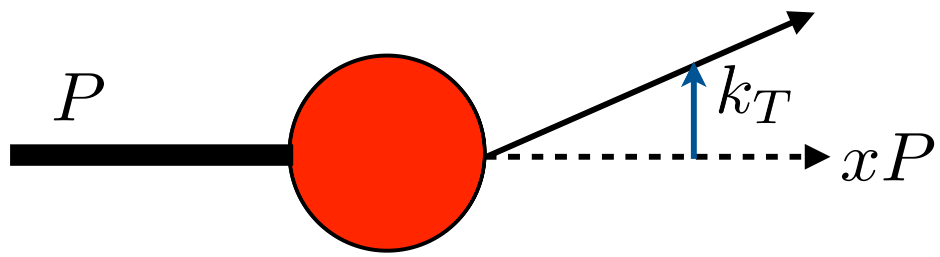
extraction of a **quark**
not collinear with the proton

bold : also collinear
red : time-reversal odd (generalized universality properties)

quark TMD PDFs

$$\Phi_{ij}(k, P; S, T) \sim \text{F.T.} \; \langle PST| \; \bar{\psi}_j(0) \; U_{[0,\xi]} \; \psi_i(\xi) \; |PST\rangle_{|_{LF}}$$

Quarks	γ^+	$\gamma^+\gamma^5$	$i\sigma^{i+}\gamma^5$
U	$\boldsymbol{f_1}$		$h_1^\perp$
L		$\boldsymbol{g_1}$	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$\boldsymbol{h_1}$, $h_{1T}^\perp$
LL	$\boldsymbol{f_{1LL}}$		$h_{1LL}^\perp$
LT	f_{1LT}	g_{1LT}	h_{1LT}, $h_{1LT}^\perp$
TT	f_{1TT}	g_{1TT}	h_{1TT}, $h_{1TT}^\perp$



extraction of a **quark**
not collinear with the proton

a similar scheme holds for
TMD FFs and gluons

bold : also collinear
red : time-reversal odd (generalized universality properties)

TMD & collinear factorization

References:
- Collins, Cambridge U. Press 2011
- SCET literature

Collinear and TMD factorization

Let's consider a process with
three separate scales:

(SIDIS, Drell-Yan, e+e- to hadrons, pp to quarkonium, ...)

hadronic mass scale

$$\boxed{\Lambda_{\rm QCD} \ll q_T \ll Q}$$

hard scale

(related to the) transverse momentum of the observed particle

The ratios $\boxed{\Lambda_{\rm QCD}/Q}$ $\boxed{\Lambda_{\rm QCD}/q_T}$ $\boxed{q_T/Q}$

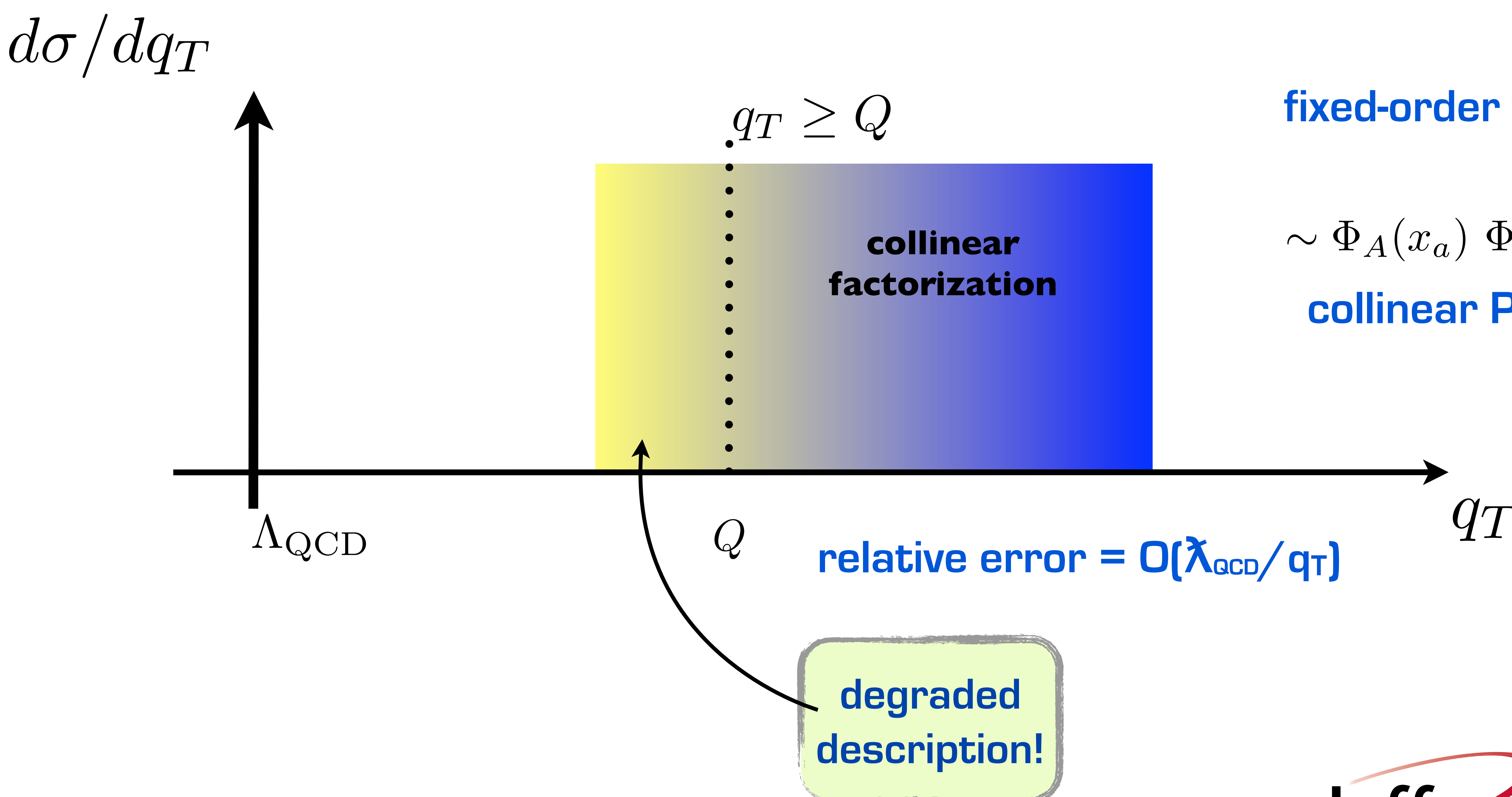
select the factorization theorem that we rely on.

According to their **values** we can access **different "projections"** of hadron structure

Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**

fixed Q, variable q_T



fixed-order term

$\sim \Phi_A(x_a)\ \Phi_B(x_b)$

collinear PDFs

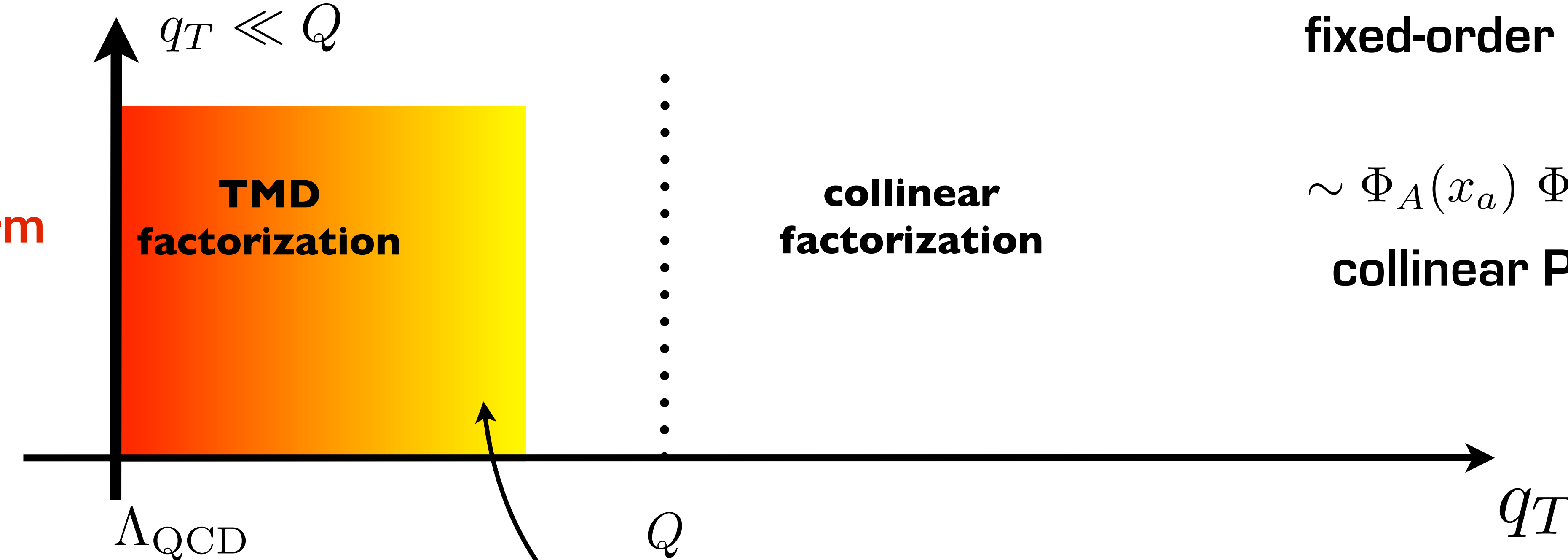
Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**

fixed Q, variable q_T

$d\sigma / dq_T$

resummed term (W)



fixed-order term

$\sim \Phi_A(x_a)\ \Phi_B(x_b)$

collinear PDFs

$\Phi_A(x_a, \mathbf{k}_{Ta})\ \Phi_B(x_b, \mathbf{k}_{Tb}) \ \sim$ **TMD PDFs**

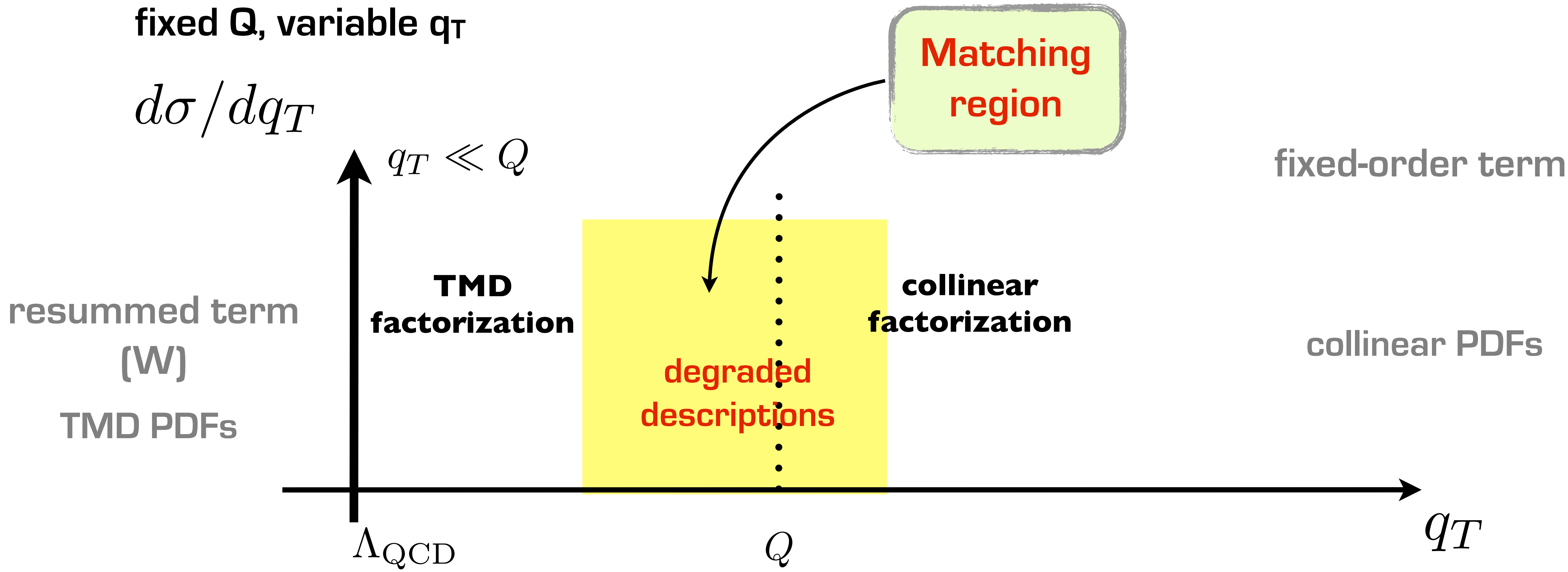
relative error = O(q_T/Q)

degraded description!

Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**

fixed Q, variable q_T



W, relative error = $O(q_T/Q)$

F.O., relative error = $O(\Lambda_{QCD}/q_T)$

We need a prescription to deal with the region where both descriptions are not good

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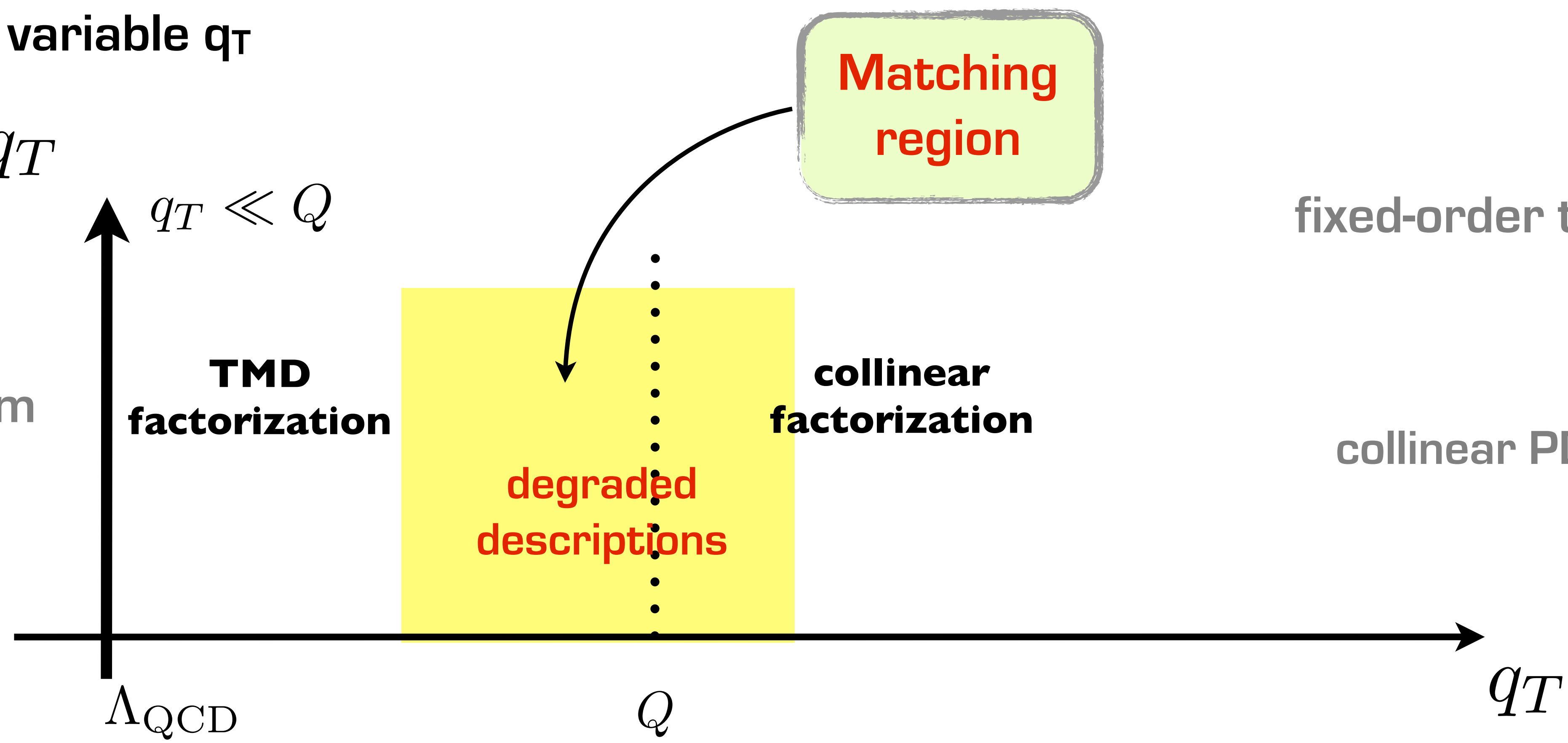
Collinear and TMD factorization

The key of phenomenology : emergence of TMD and collinear distributions from **factorization theorems**

fixed Q, variable q_T

$d\sigma/dq_T$



fixed-order term

resummed term (W)

TMD PDFs

collinear PDFs

The extraction of the **nonperturbative (NP) part of TMDs** is affected by the **description of the whole q_T range**

- Crucial, especially at **low Q** (e.g. JLab kinematics), where the **regions shrink**
- Pheno: we are making progress
- polarization ?

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W-term & TMDs

W-term : transverse momentum resummation in terms of TMDs

$$W(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} \; e^{i q_T \cdot b_T} \; \tilde{W}(b_T, Q)$$

b_T is the Fourier-conjugated variable of the (partonic and observed) transverse momenta

$$\tilde{W}(b_T, Q) \sim \tilde{F}_i^{h_1}(x_1, b_T; \mu, \zeta_1) \; \tilde{F}_j^{h_2}(x_2, b_T; \mu, \zeta_2)$$

Product of Fourier-transformed TMDs

TMD evolution is multiplicative in b_T space

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W-term & TMDs

W-term : transverse momentum resummation in terms of TMDs

$$W(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} \; e^{i q_T \cdot b_T} \; \tilde{W}(b_T, Q)$$

b_T is the Fourier-conjugated variable of the (partonic and observed) transverse momenta

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Need a regularization to recover collinear factorization upon integration over qT:

$$b_T \to \bar{b}_T \geq b_{\min} \sim 1/Q \qquad \Longrightarrow \qquad \int d^2 q_T \; W(q_T, Q) \sim f_i^{h_1}(x_1; \mu) \; f_j^{h_2}(x_2; \mu)$$

Collins et al. PRD94 2016

$$\tilde{W}(b_T, Q) \sim \tilde{F}_i^{h_1}(x_1, b_T; \mu, \zeta_1) \; \tilde{F}_j^{h_2}(x_2, b_T; \mu, \zeta_2)$$

Product of Fourier-transformed TMDs

TMD evolution is multiplicative in b_T space

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W-term & TMDs

FT of TMDs :

$$\tilde{F}_i(x, b_T; Q, Q^2) = \tilde{F}_i(x, b_T, \mu_{\hat{b}}, \mu_{\hat{b}}^2) \times$$

$$\exp\left\{ \int_{\mu_{\hat{b}}}^{Q} \frac{d\mu}{\mu} \gamma_F[\alpha_s(\mu), Q^2/\mu^2] \right\} \left(\frac{Q^2}{\mu_{\hat{b}}^2} \right)^{-K(\hat{b}_T;\mu_{\hat{b}}) - g_K(\overline{b_T};\{\lambda\})}$$

Sudakov form factor : perturbative and **nonperturbative** contributions

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Collinear distribution!

(input) TMD distribution : Wilson coefficients and **intrinsic part**

$$\tilde{F}_i(x, b_T; \mu_{\hat{b}}, \mu_{\hat{b}}^2) = \sum_{j=q,\bar{q},g} C_{i/j}(x, \hat{b}_T; \mu_{\hat{b}}, \mu_{\hat{b}}^2) \otimes f_j(x; \mu_{\hat{b}}) \tilde{F}_{i,NP}(x, \overline{b}_T; \{\lambda\})$$

Nonperturbative parts defined in a "negative" way : **observed-calculable**

W-term and TMDs

Distribution for intrinsic transverse momentum (and its FT):

$$\boxed{\tilde{F}_{i,NP}(x,\bar{b}_T;\{\lambda\})}$$

a Gaussian ?

Soft gluon emission

$$\boxed{g_K(\bar{b}_T;\{\lambda\})}$$

W-term and TMDs

Distribution for intrinsic transverse momentum (and its FT):

$$\boxed{\tilde{F}_{i,NP}(x, \bar{b}_T; \{\lambda\})}$$

a Gaussian ?

Soft gluon emission

$$\boxed{g_K(\bar{b}_T; \{\lambda\})}$$

Separation of **b_T regions**

$$\boxed{\hat{b}_T(b_T; b_{\min}, b_{\max})} \begin{array}{l} \nearrow \; b_{\max} \;, \quad b_T \to +\infty \\ \sim \;\; b_T \;, \quad b_{\min} \ll b_T \ll b_{\max} \\ \searrow \; b_{\min} \;, \quad b_T \to 0 \end{array}$$

High b_T limit : avoid Landau pole

Low b_T limit : recover fixed order expression

W+Y

References:
- Collins-Soper-Sterman, NPB 1985
- Arnold-Kauffman, NPB 1991
- Collins, Cambridge U. Press 2011

Philosophy

W : good description of the cross section when
$q_T \ll Q$;
its relative error is $O(q_T/Q)$

A term $O(q_T/Q)$ is needed - we call it **Y- to bridge the two descriptions**; the corrections need to vanish where we know that W and F.O. are good approximations of the cross section

Collinear fixed-order (F.O.) calculation : good description of the cross section when $q_T \gtrsim Q$;
its relative error is $O(\lambda_{QCD}/q_T)$

W+Y explained

Cross section

$$\Gamma = \mathbf{T}_{\mathrm{TMD}}\Gamma + [\Gamma - \mathbf{T}_{\mathrm{TMD}}\Gamma]$$

$$\approx \underbrace{\mathbf{T}_{\mathrm{TMD}}\Gamma}_{\mathbf{W}} + \underbrace{\mathbf{T}_{\mathrm{coll}}[\Gamma - \mathbf{T}_{\mathrm{TMD}}\Gamma]}_{\mathbf{Y}}$$

Y = Fixed Order (**F.O.**) - F.O. expansion of resummed (**ASY**)

$$[\Gamma - \mathbf{T}_{\mathrm{TMD}}\Gamma] = \boxed{O\left(\frac{q_T}{Q}\right)^a \Gamma}$$

(error at low q_T)

$$\mathbf{T}_{\mathrm{coll}}[\Gamma - \mathbf{T}_{\mathrm{TMD}}\Gamma] = [\Gamma - \mathbf{T}_{\mathrm{TMD}}\Gamma]\,[1 + \boxed{O\left(\frac{m}{q_T}\right)^b}\,]$$

(error at higher q_T)

$$= [\Gamma - \mathbf{T}_{\mathrm{TMD}}\Gamma] + O\left(\frac{q_T}{Q}\right)^a O\left(\frac{m}{q_T}\right)^b \Gamma$$

$$= [\Gamma - \mathbf{T}_{\mathrm{TMD}}\Gamma] + \underbrace{\boxed{O\left(\frac{m}{Q}\right)^{min(a,b)} \Gamma}}_{\text{error for } \mathbf{W+Y}}$$

"global" error

$d\sigma = W + Y + O(\lambda_{QCD}/Q)^c\, d\sigma$

from **B. Wang** - GHP workshop 2017

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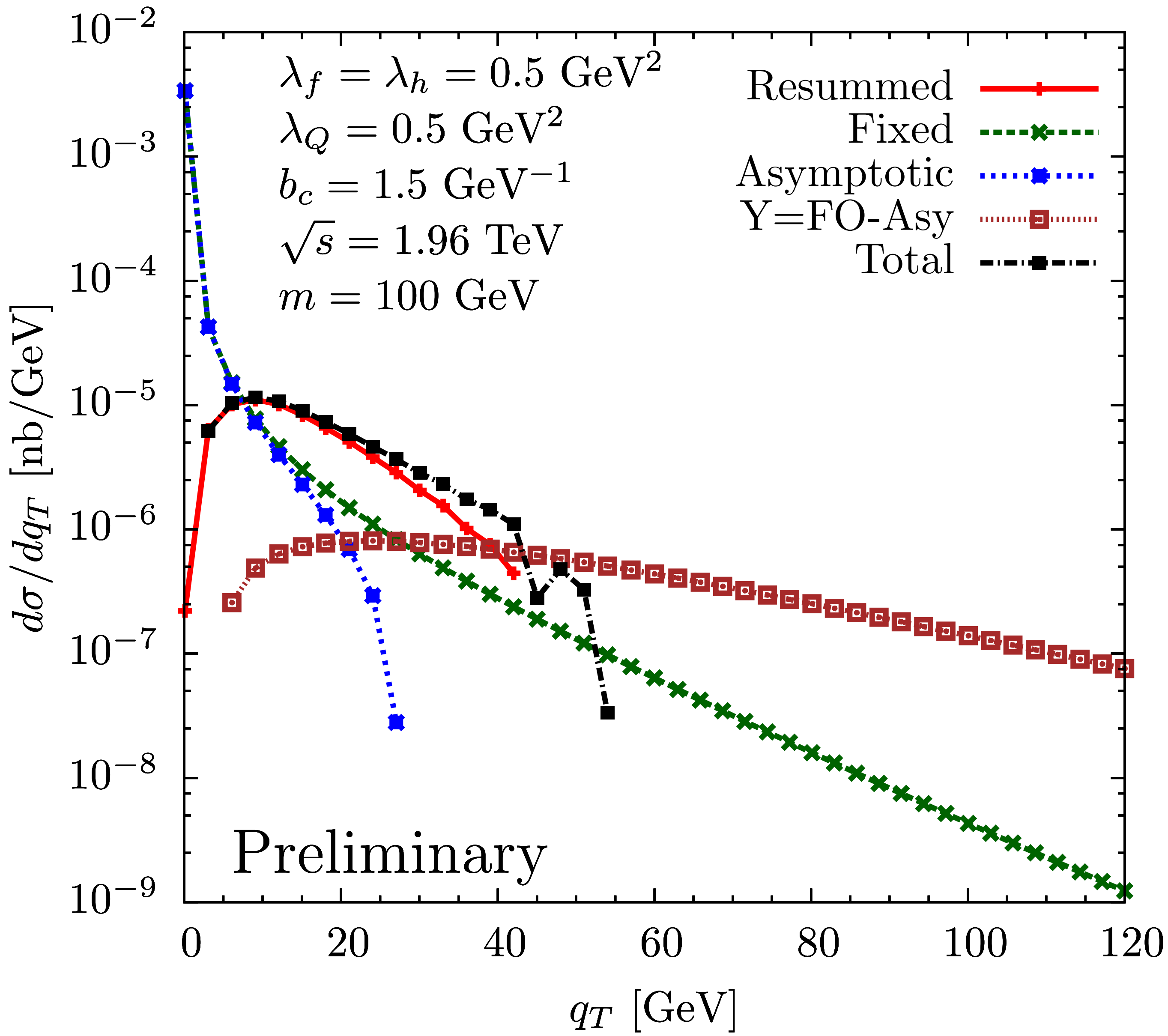
W+Y at work - pp @ high Q (fake η)

AS - PhD thesis

pp at high Q

fake state with M=100 GeV



similar behavior observed in Boglione et al. JHEP2015 for
SIDIS @ Q2=5*10^3 GeV2, and 100GeV2 (HERA kinematics)

FO : $1/q_T^2$

ASY : $1/q_T^2 * \ln Q^2/q_T^2$

Y=FO-ASY should be negligible at low q_T:
yes, because the errors on ASY(W) are under control since Q is high

Y should approach FO at high q_T:
no, because ASY(W) gets <0 and Y>FO

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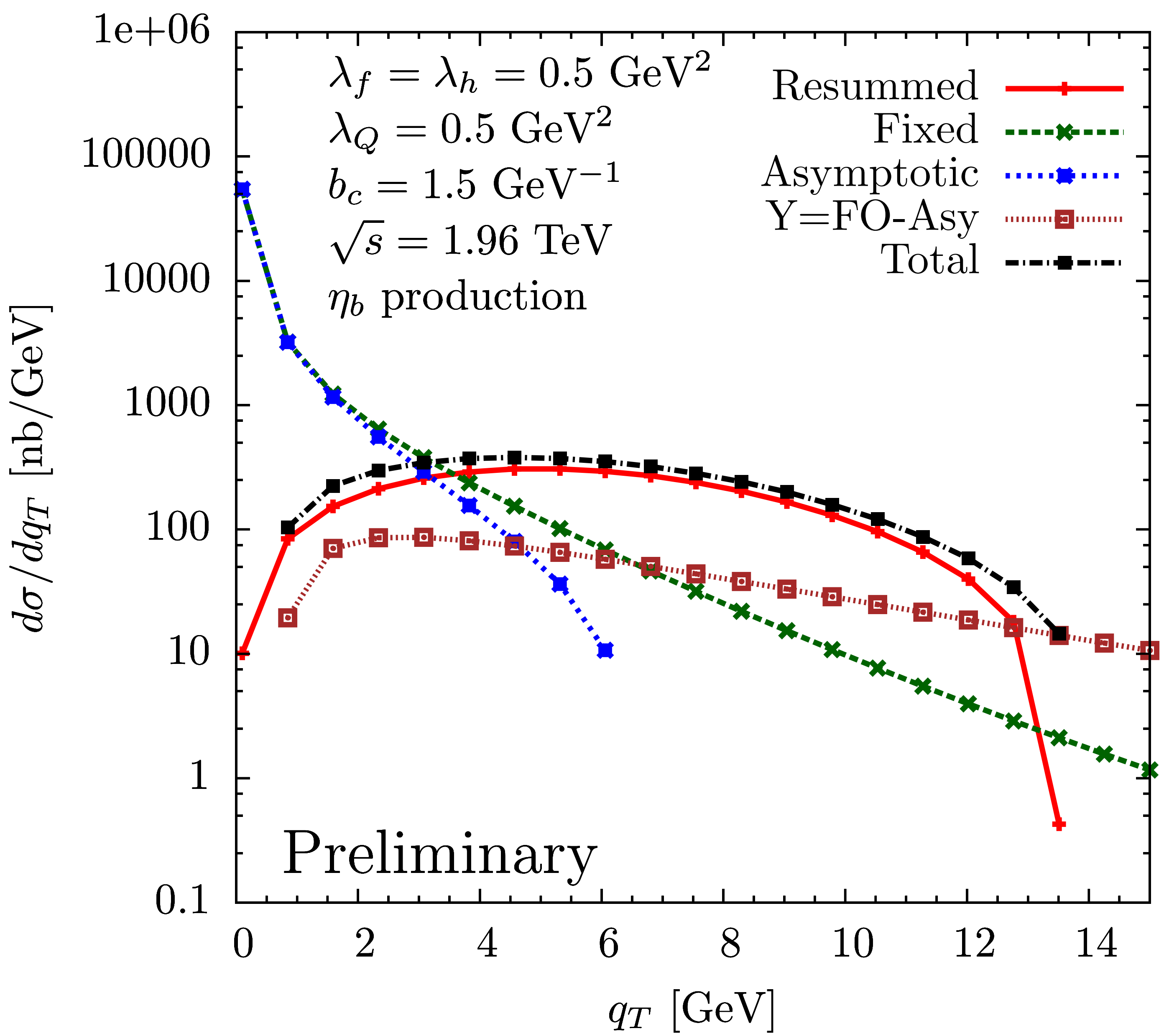
W+Y at work - pp @ moderate Q (η_b)

AS - PhD thesis

pp at moderate Q

$M(\eta_b)$=9.39 GeV



FO : $1/q_T^2$

ASY : $1/q_T^2 * \ln Q^2/q_T^2$

Y=FO-ASY should be negligible at low q_T:
no, it's 10%; the cancellation might not work properly due to the high relative error for the ASY(W) at low q_T;
also, W dominates the total also outside its validity region

Y should approach FO at high q_T:
no, because ASY gets <0 and Y>FO

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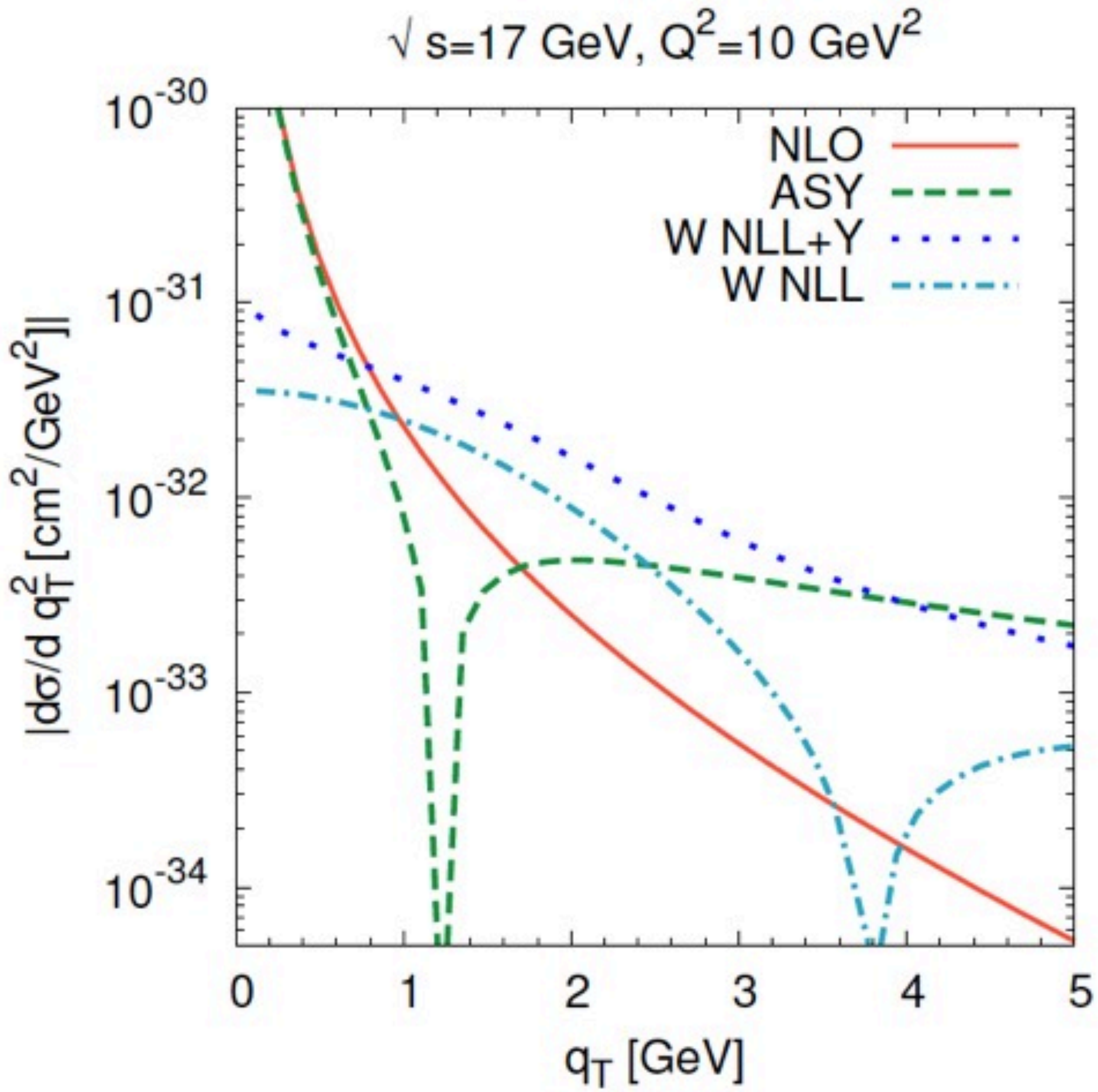
W+Y at work - SIDIS @ low Q (Compass)

Boglione et al., JHEP 2015

SIDIS at low Q

Compass/JLab kin.



FO : $1/q_T^2$

ASY : $1/q_T^2 * \ln Q^2/q_T^2$

Lowering again Q, we observe **the same trend** as before: Y non-negligible at low q_T and Y that does not converge to FO (because ASY(W) < 0)

problems with W+Y

low q_T : for moderate and small Q values, Y significantly contributes to the cross section, even if that should be dominated by W

high q_T : for all Q values, since ASY(W) does not vanish at high q_T, Y does not converge to F.O.

Problems manifest **especially when the hard scale Q is low**, when the TMD and collinear regions are not well separated

We need **improvements** and/or **alternative approaches**

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Improved W+Y

References:
- J.C. Collins, L. Gamberg, A. Prokudin, N. Sato, T.C. Rogers, B. Wang, PRD94 2016

see Leonard Gamberg's talk today

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Improved W+Y

low q_T : for moderate and small Q values, Y significantly contributes to the cross section, even if that should be dominated by W ⟵ **the problem**

$$\mathbf{T}_{\text{coll}}\left[\Gamma - \overline{\mathbf{T}_{\text{TMD}}}\Gamma\right] = \left[\Gamma - \overline{\mathbf{T}_{\text{TMD}}}\Gamma\right]\left[1 + O\left(\frac{m}{q_T}\right)^{b} + O\left(\frac{m}{q_T}\right)^{b'} + \ldots\right]$$

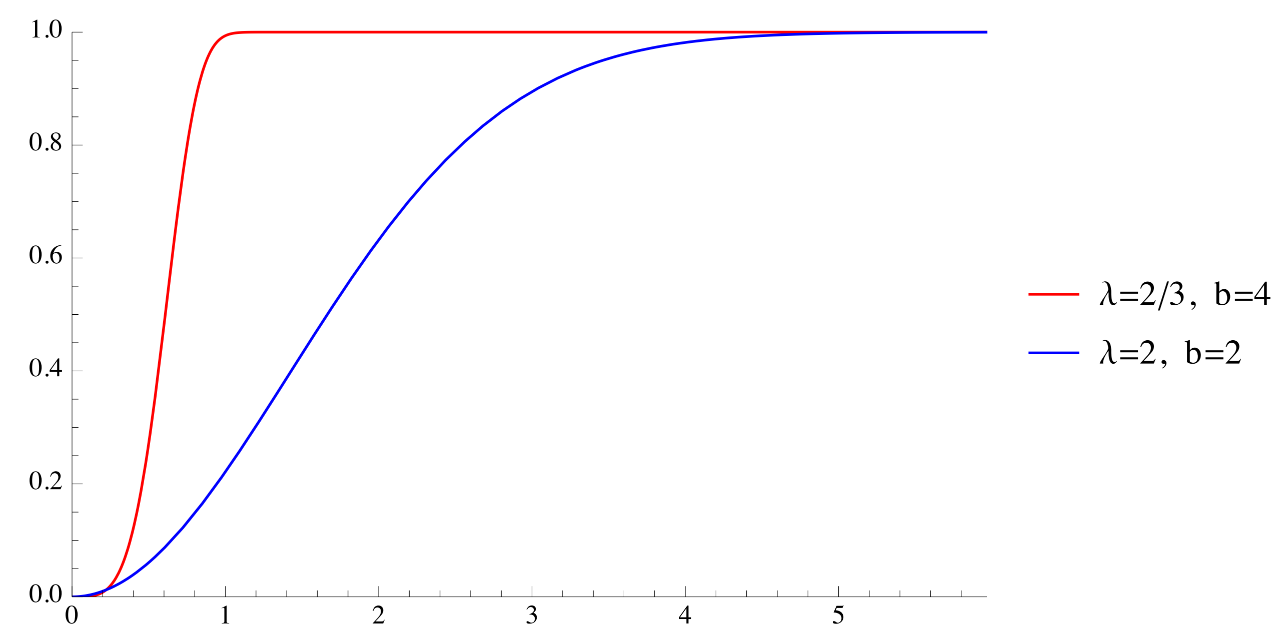
the corrections dominate at low q_T; a possibility is to introduce a low-q_T cutoff for Y:

$$Y(q_T) \rightarrow \boxed{X(q_T/\lambda, b)}\, Y(q_T)$$

cutoff function at low q_T:

$$X(q_T/\lambda, b) = 1 - \exp\left[-(q_T/\lambda)^b\right]$$

λ controls how "early" in q_T the cutoff starts
($q_T < \lambda$: nonperturbative region)
b controls the steepness of the cutoff



see also **B. Wang** (GHP 2017) and L. Gamberg's talk

Improved W+Y

high q_T : for all Q values, since ASY(W) does not vanish at high q_T, Y does not converge to F.O. ⟵ **the problem**

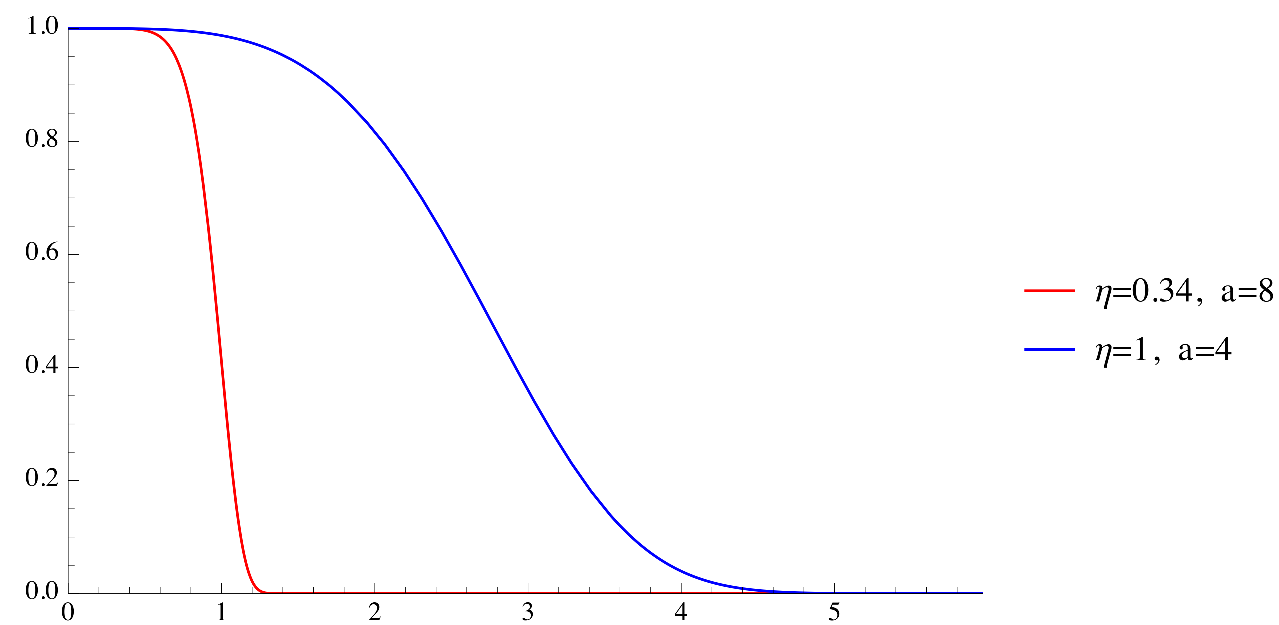
a possibility is to introduce a high-q_T cutoff for W (implies for ASY too) :

$$W_{\rm new}(q_T, Q) = \boxed{\Xi(q_T/\eta Q, a)} \int \frac{d^2 b_T}{(2\pi)^2} \, e^{i q_T \cdot b_T} \, \tilde{W}(\bar{b}_T, Q)$$

cutoff function at high q_T:

$$\Xi(q_T/\eta Q, a) = \exp\big[- (q_T/\eta Q)^a \big]$$

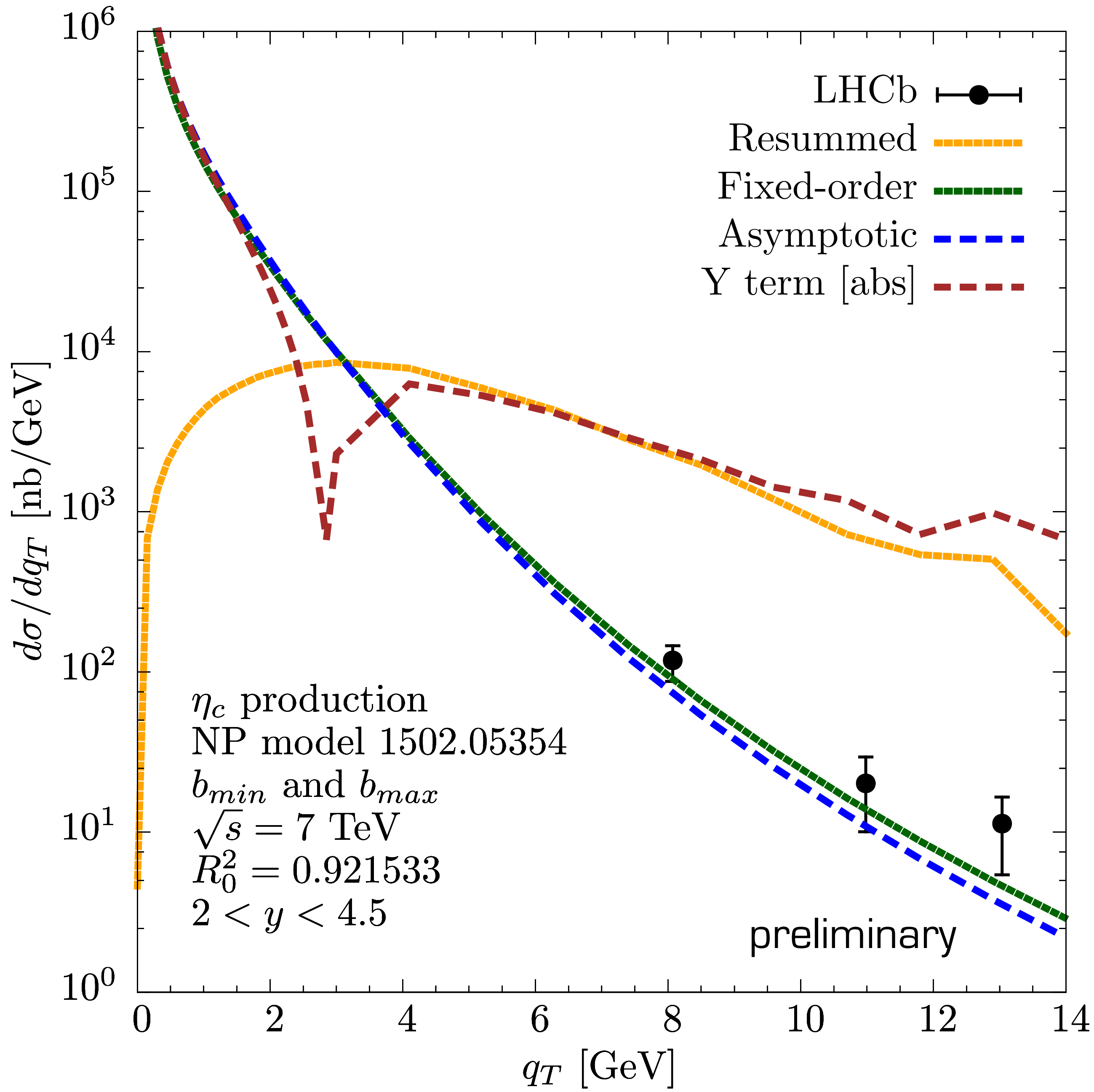
η controls how "early" in q_T the cutoff starts
a controls the steepness of the cutoff



see also **B. Wang** (GHP 2017) and L. Gamberg's talk

improved W+Y at work - η_c @ LHC

"bare" contributions - no damping



low-energy process
$Q = M(\eta_c) = 2.98$ GeV

b_{min} and b_{max} prescriptions

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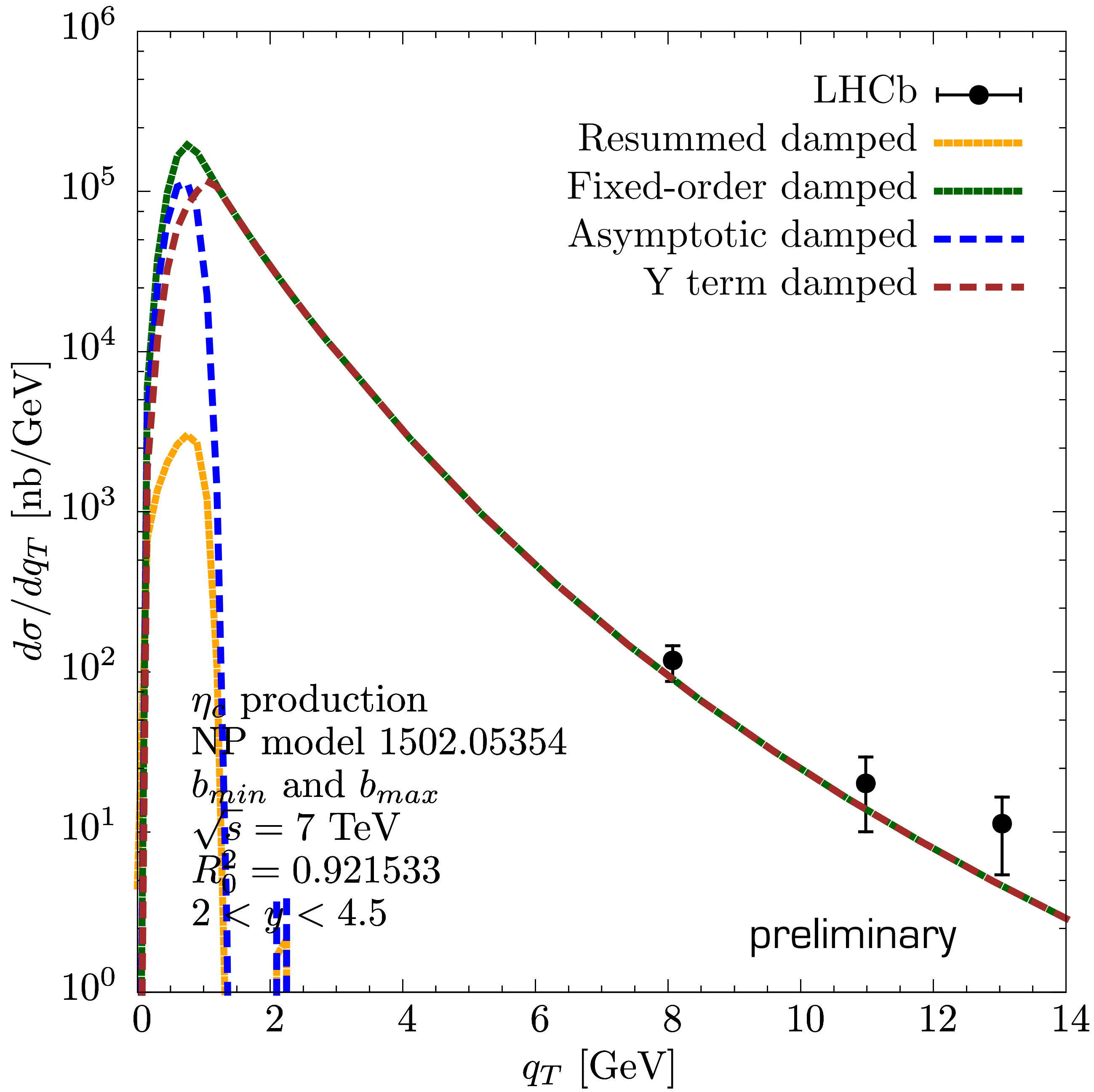
improved W+Y at work - η_c @ LHC

damped contributions



low-energy process
Q = M(η_c) = 2.98 GeV

b_{min} and b_{max} prescriptions

W : damped at high q_T
FO : damped at low q_T
ASY : damped at low q_T
and at high q_T (via W)

damping parameter as in PRD94 '16
(optimized for SIDIS @ low Q) :
- Y : λ=2/3, b=4
- W : η=0.34, a=8

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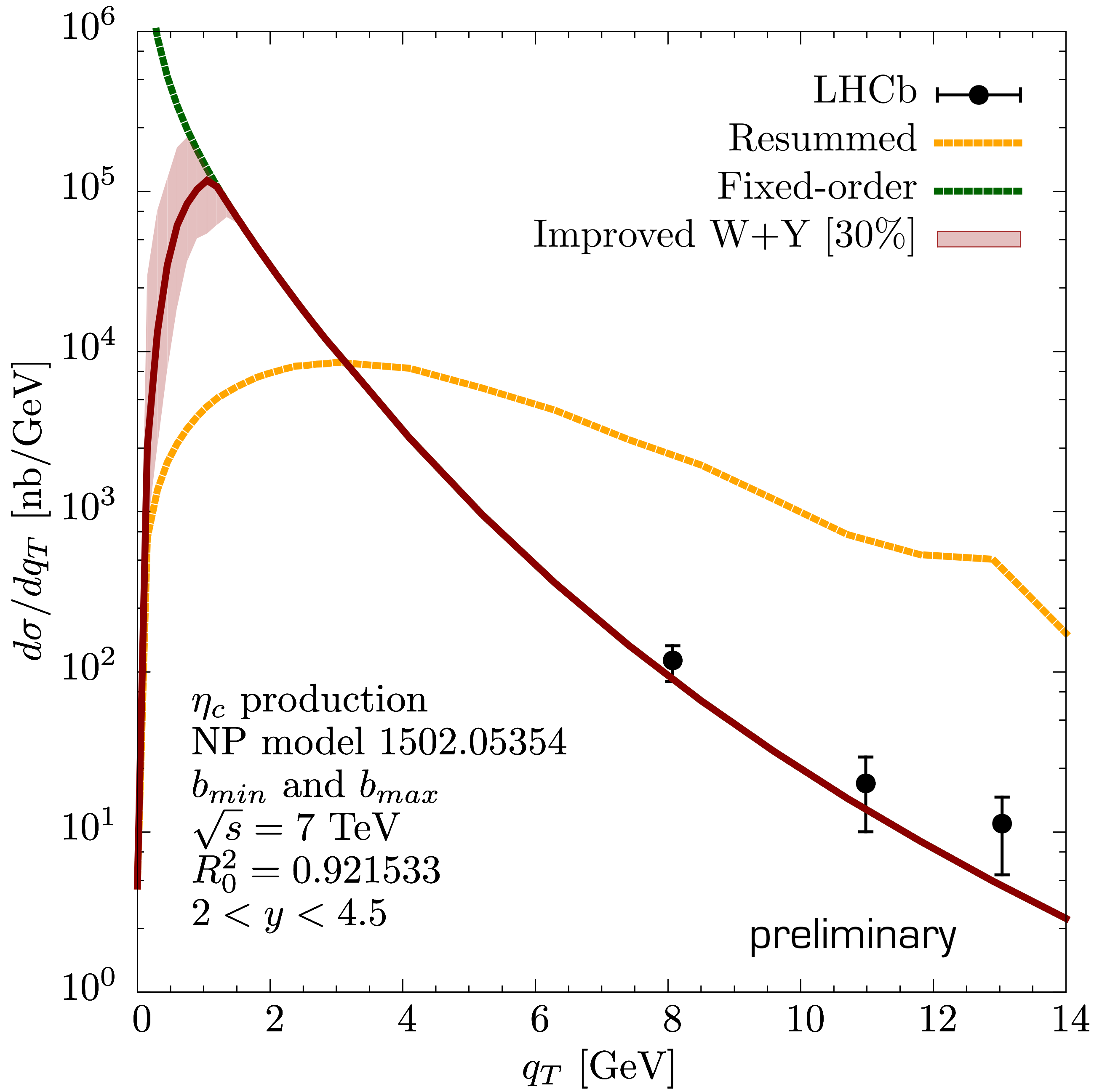
improved W+Y at work - η_c @ LHC

matched contributions



low-energy process
Q = M(η_c) = 2.98 GeV

b_{min} and b_{max} prescriptions

the cross section matches very rapidly to the FO

estimate of the uncertainty on the matching :
red band = **vary** by **30%**
the 4 parameters in the cutoff functions and take the envelope

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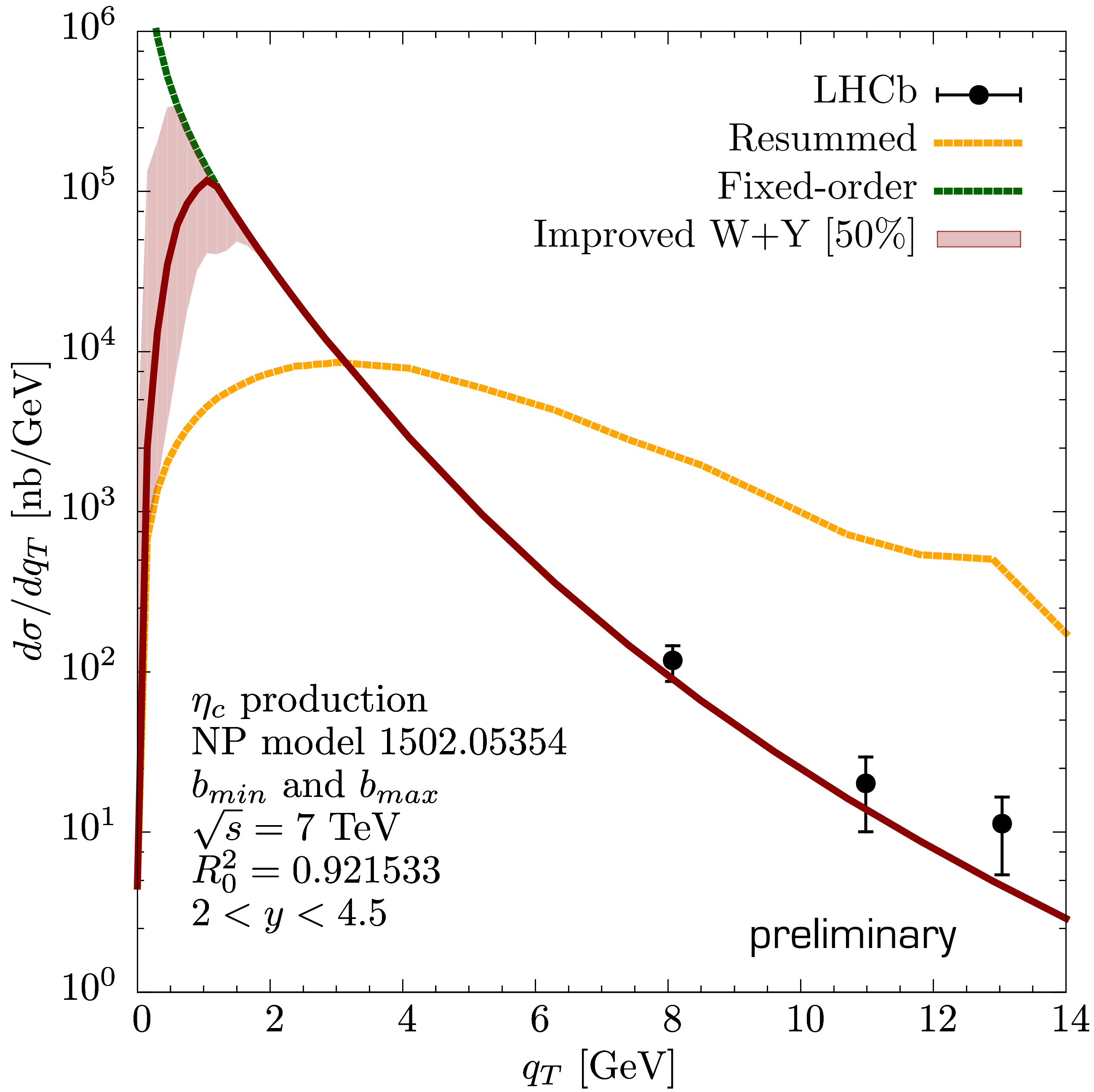
improved W+Y at work - η_c @ LHC

matched contributions



low-energy process
Q = M(η_c) = 2.98 GeV

b_{min} and b_{max} prescriptions

the cross section matches very rapidly to the FO

estimate of the uncertainty on the matching :
red band = **vary** by **50%**
the 4 parameters in the cutoff functions and take the envelope

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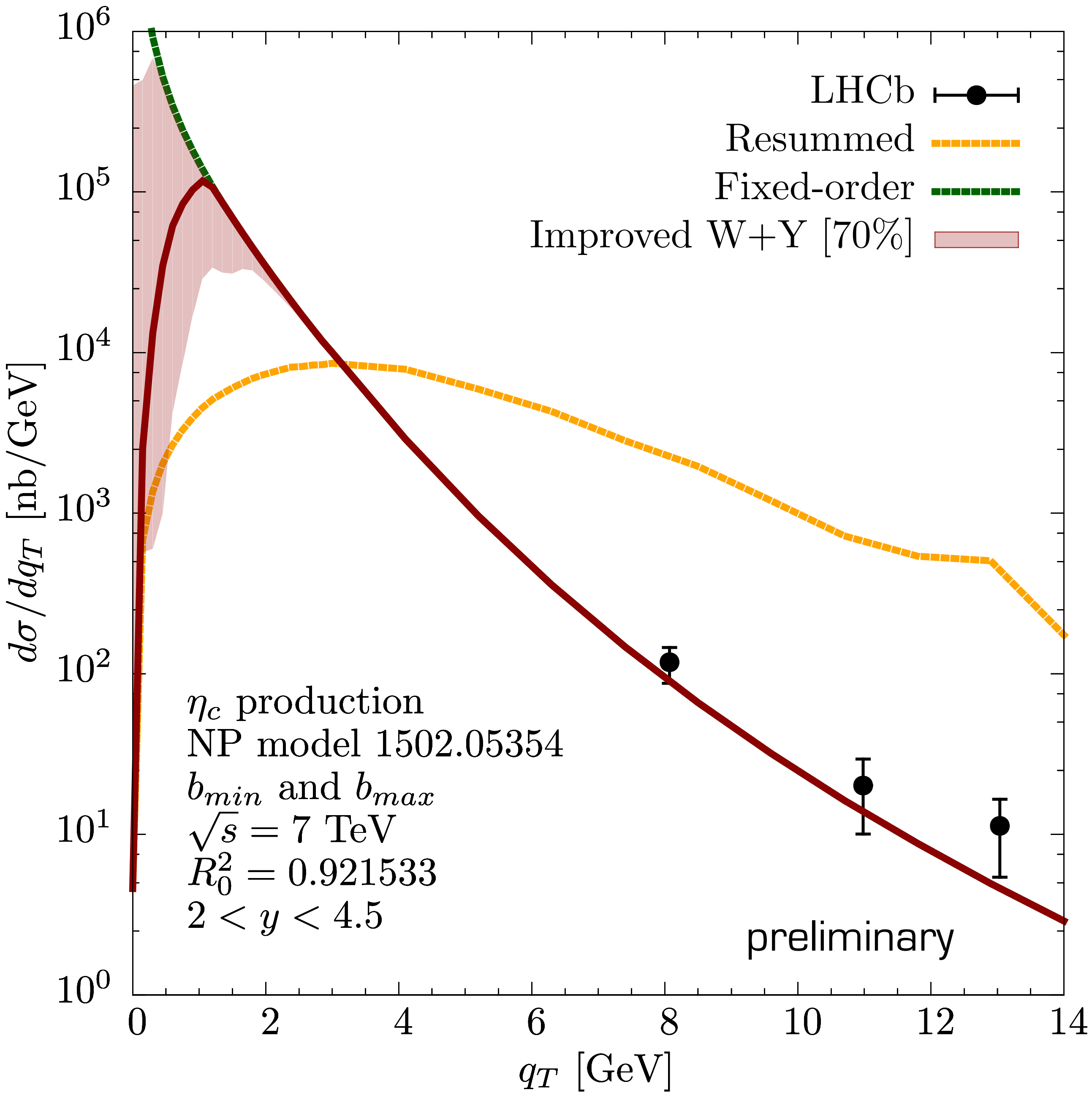
improved W+Y at work - η_c @ LHC

matched contributions



low-energy process
Q = M(η_c) = 2.98 GeV

b_{min} and b_{max} prescriptions

the cross section matches very rapidly to the FO

estimate of the uncertainty on the matching :
red band = **vary** by **70%**
the 4 parameters in the cutoff functions and take the envelope

the uncertainty is not small -
the implementation of the damping
highly affects the matching

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The log-average scheme

References:
- M.G. Echevarria, T. Kasemets, J.P. Lansberg, C. Pisano, AS - in preparation

“Averaging” TMD and collinear fact.

We propose a **new way of matching** the resummed term and the fixed-order calculation, based on a weighted average of the two contributions.

The **weights** involve the **power corrections to the factorized expression** at low and high q_T

our (**conservative**) choice for the errors:

at low q_T : $\sigma = W\,(1 + (q_T/Q)^m)$

at high q_T : $\sigma = FO\,(1 + (\lambda/q_T)^n)$

$$\Delta W = (q_T/Q)\, W$$

$$\Delta FO = (\lambda/q_T)\, FO$$

Average ...

The weighted
average is a commonly accepted
quantity to report as the best estimate
for the value of a measured quantity

$$\bar{\sigma} = \frac{\omega_1 \ \sigma_1 + \omega_2 \ \sigma_2}{\omega_1 + \omega_2}$$

Average ...

The weighted
average is a commonly accepted
quantity to report as the best estimate
for the value of a measured quantity

$$\bar{\sigma} = \frac{\omega_1 \ \sigma_1 + \omega_2 \ \sigma_2}{\omega_1 + \omega_2}$$

But **when the errors are proportional to the measurements**, the weighted average is **skewed towards the lower values**!

Example :

$\sigma_1 = 1.20 \pm 0.24$

$\sigma_2 = 0.80 \pm 0.16$

non-weighted average = 1.00

weighted average = 0.92
(closer to the lower value)

same relative error = 0.2

Average ...

The weighted average is a commonly accepted quantity to report as the best estimate for the value of a measured quantity

$$\bar{\sigma} = \frac{\omega_1 \ \sigma_1 + \omega_2 \ \sigma_2}{\omega_1 + \omega_2}$$

But **when the errors are proportional to the measurements**, the weighted average is **skewed towards the lower values**!

If we match W and FO calculating their weighted average (using ΔW and ΔFO) the cross section is **closer to the W** in the matching region

Example :

$\sigma_1 = 1.20 \pm 0.24$

$\sigma_2 = 0.80 \pm 0.16$

non-weighted average = 1.00

weighted average = 0.92
(closer to the lower value)

same relative error = 0.2

… and log-average

The **logarithmic weighted average (log-average)** is **less skewed towards the lower values**, when the errors are proportional to the measurements

$$\overline{\log\sigma} = \frac{\omega_1 \ \log\sigma_1 + \omega_2 \ \log\sigma_2}{\omega_1 + \omega_2}$$

$$\omega_i = (\Delta\log\sigma_i)^{-2} \qquad \Delta\log\sigma_i = \frac{\Delta\sigma_i}{\sigma_i}\frac{1}{\ln 10}$$

Example :

σ_1 = 1.20 ± 0.24

σ_2 = 0.80 ± 0.16

same relative error = 0.2

non-weighted average = 1.00

weighted average = 0.92

log-average = 0.98
(closer to the non-w. average)

… and log-average

The **logarithmic weighted average (log-average)** is **less skewed towards the lower values**, when the errors are proportional to the measurements

$$\overline{\log \sigma} = \frac{\omega_1 \ \log \sigma_1 + \omega_2 \ \log \sigma_2}{\omega_1 + \omega_2}$$

$$\omega_i = (\Delta \log \sigma_i)^{-2} \qquad \Delta \log \sigma_i = \frac{\Delta \sigma_i}{\sigma_i} \frac{1}{\ln 10} \qquad \Delta(\overline{\log \sigma}) = (\omega_1 + \omega_2)^{-1/2}$$

we implement the **matching of W and FO as the log-average** of σ_1 = W and σ_2 = FO

uncertainty on the cross section in the matching region from error propagation (**blue band** in next slides)

the matching is given by:

$$10^{\overline{\log \sigma}} \pm 10^{\overline{\log \sigma}} \ \ln 10 \ \Delta\left(\overline{\log \sigma}\right)$$

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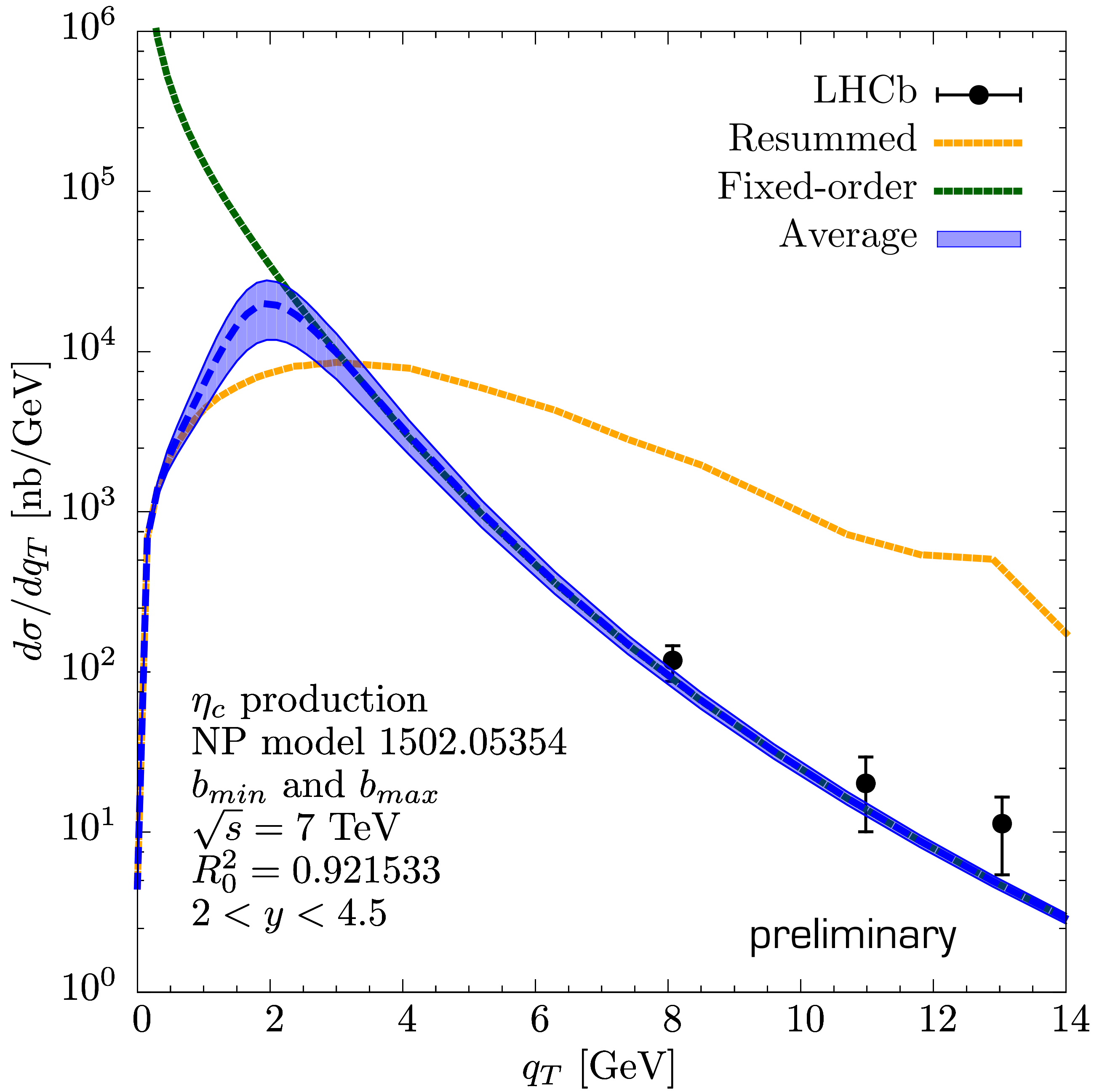
Log-average at work - η_c @ LHC

matched contributions



low-energy process
$Q = M(\eta_c) = 2.98$ GeV

b_{min} and b_{max} prescriptions

smooth matching from W to FO

- W dominates for $q_T < 1$ GeV
- FO dominates for $q_T > 3$ GeV

blue band: uncertainty from matching

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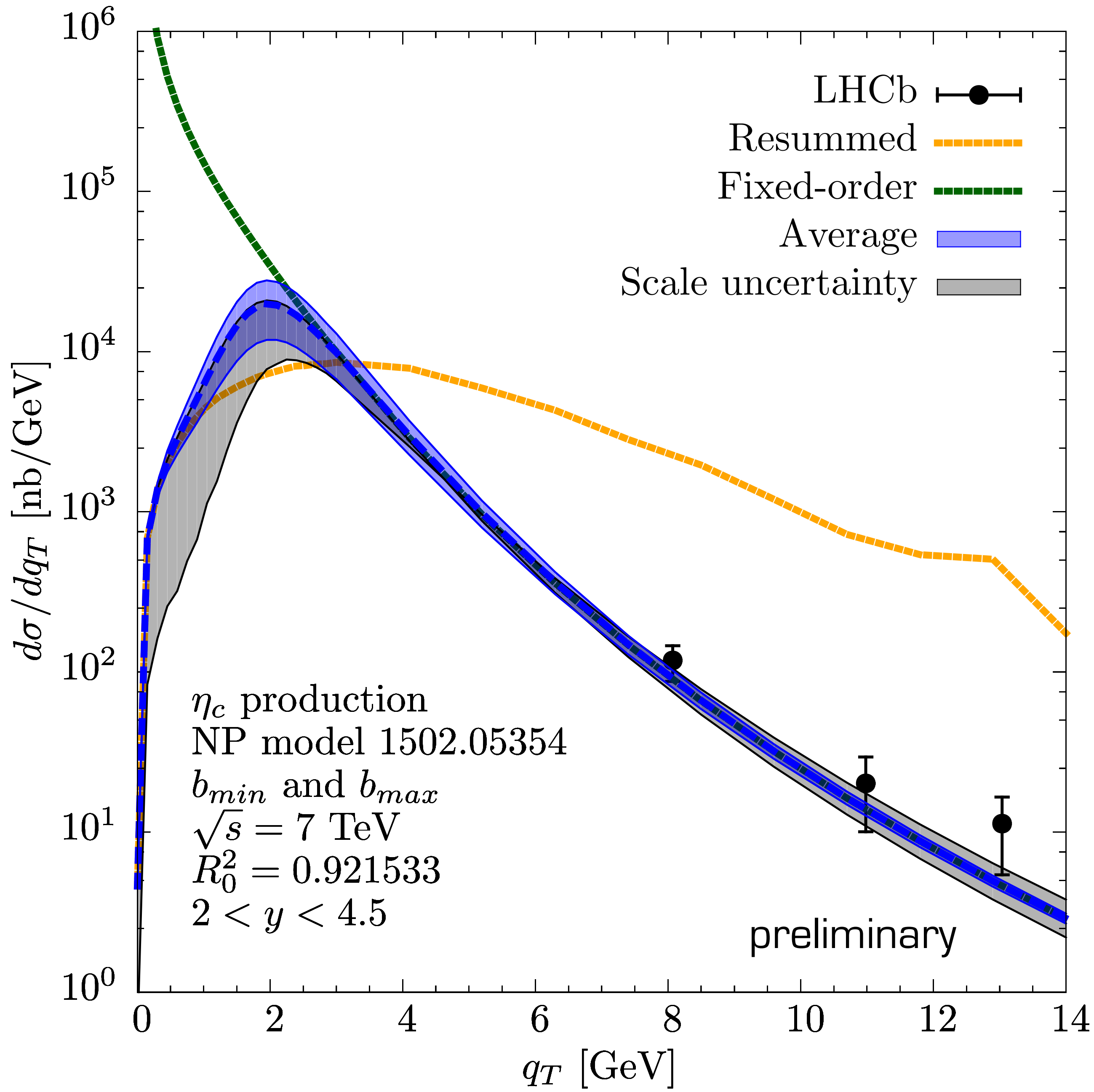
Log-average at work - η_c @ LHC

matched contributions



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blue band: uncertainty from matching

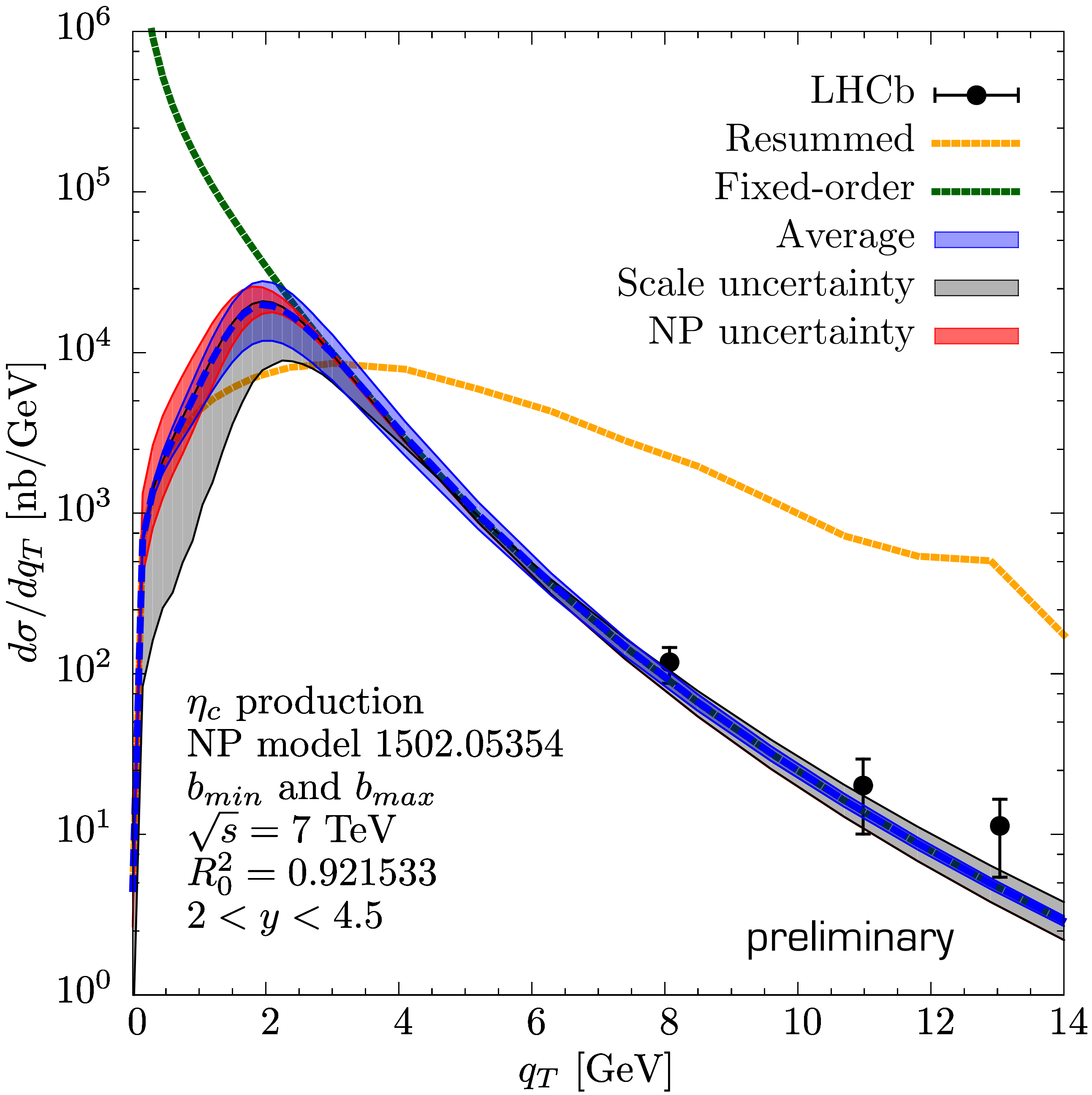
grey band: scale uncertainty

$$\mu_i^2 = \zeta_i = \mu_{\hat{b}}^2 \ , \quad \mu_{F.O.} = m_T$$

fact. 2 variation and envelope

Log-average at work - η_c @ LHC

matched contributions



low-energy process
Q = M(η_c) = 2.98 GeV

b_{min} and b_{max} prescriptions

smooth matching from W to FO

- W dominates for $q_T < 1$ GeV
- FO dominates for $q_T > 3$ GeV

blue band: uncertainty from matching

grey band: scale uncertainty

red band: nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = -\left[\frac{a_1}{2} + \frac{a_2}{2}\ \ln Q^2\right]\ \bar{b}_T^2$$

$a_i = 0.5$ GeV^2, var. 50%, envelope

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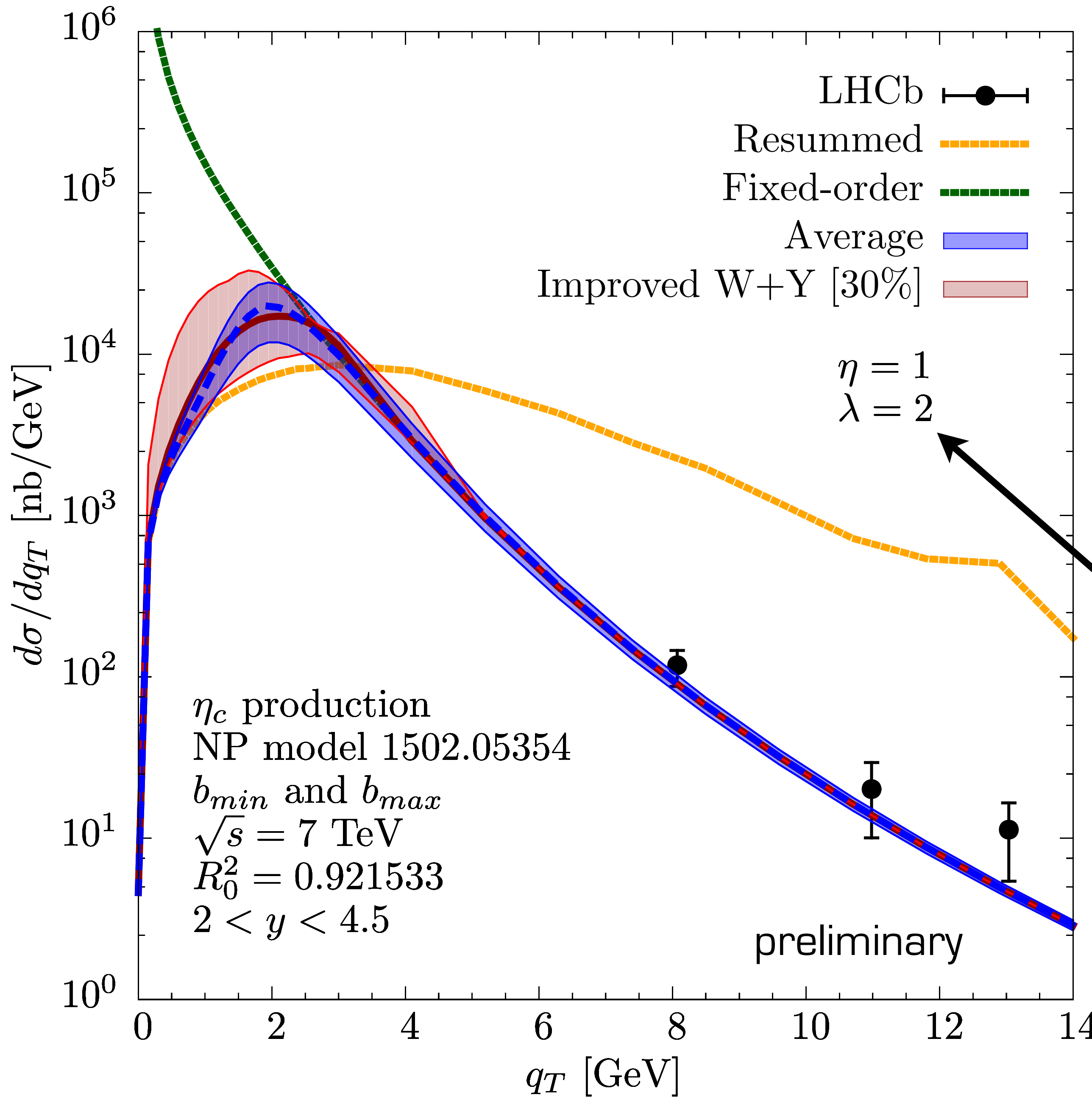
Log-average at work - η_c @ LHC

log-average vs improved W+Y



different!

low-energy process
Q = M(η_c) = 2.98 GeV

b_{min} and b_{max} prescriptions

smooth matching from W to FO

- W dominates for q_T < 1 GeV
- FO dominates for q_T > 3 GeV

blue band: uncertainty from log-average matching

red band: uncertainty from improved W+Y matching (larger)

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Conclusions and future developments

- TMD and collinear factorization need to be matched in the intermediate-q_T region where their relative errors are large

- the **standard W+Y** scheme is **problematic**, especially at low Q (where the bulk of experimental data currently is!)

- in the **improved W+Y** scheme the problems are solved by **manually damping W and Y** at high and low q_T respectively; **the way the damping is performed matters!**

- we present an **alternative scheme**, that matches TMD and collinear factorization calculating a **logarithmic-weighted average of W and FO**

Conclusions and future developments

- we present an **alternative scheme**, that matches TMD and collinear factorization calculating a **logarithmic-weighted average of W and FO**:

= **very simple idea but not less accurate than W+Y**: the errors damp the contributions outside their validity region

= there is **no need to calculate the ASY** term; **no cancellations needed**

= it is based on the knowledge of the **power corrections to W and FO**

= there is **no need** of **cutoff functions** or phenomenological **parameters**

= the prescription works well **from low to high Q**

= to-be-tested in other processes (e.g. **SIDIS** at Compass-JLab kinematics)

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Backup

Hadron structure in 3D momentum space

TMD correlators | **distributions**

quark content

$$\Phi_{ij}(x, \mathbf{k}_T) \sim \text{F.T.}\ \langle P|\bar{\psi}_j(0)\ U_{[0,\xi]}\ \psi_i(\xi)|P\rangle_{|_{\xi^+=0}} \longrightarrow$$

quark TMD PDFs

Bacchetta, Mulders
Phys.Rev. D62 (2000) 114004

gluon content

$$\Gamma^{\alpha\beta}(x, \mathbf{k}_T) \sim \text{F.T.}\ \langle P|F^{+\alpha}(0)\ U_{[0,\xi]}\ F^{+\beta}(\xi)\ U'_{[\xi,0]}|P\rangle_{|_{\xi^+=0}} \longrightarrow$$

gluon TMD PDFs

Boer, Cotogno, van Daal, Mulders, Signori, Zhou
JHEP 1610 (2016) 013

loop correlator

$$\Gamma_0(x, \mathbf{k}_T) \sim \delta(x)\ \text{F.T.}\ \langle P|U^{[+]}_{[0,\xi]}\ U^{[-]}_{[\xi,0]}|P\rangle_{|_{\xi^+=0}} \longrightarrow$$

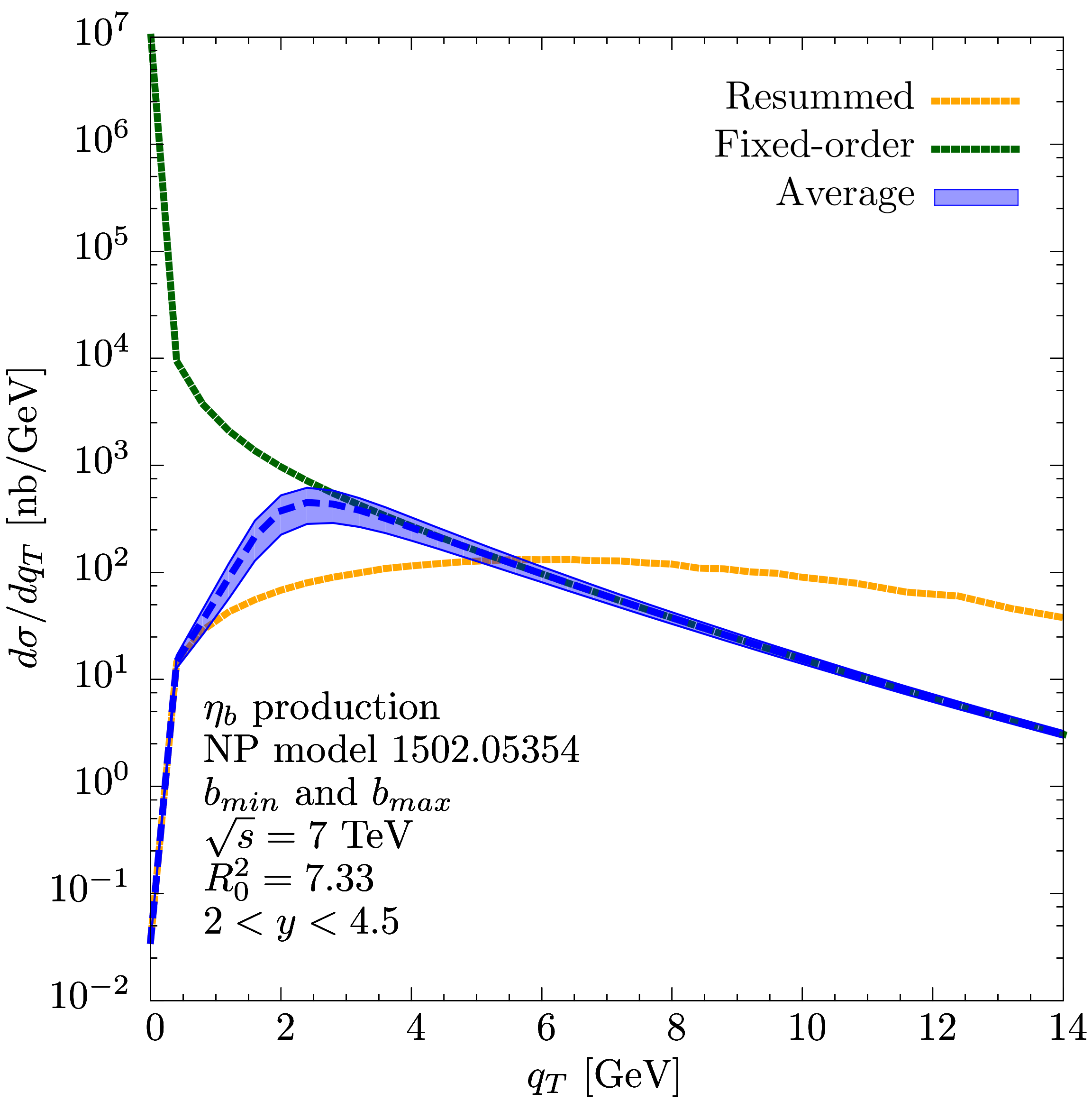
Wilson loop TMD PDFs

Boer, Cotogno, van Daal, Mulders, Signori, Zhou
JHEP 1610 (2016) 013

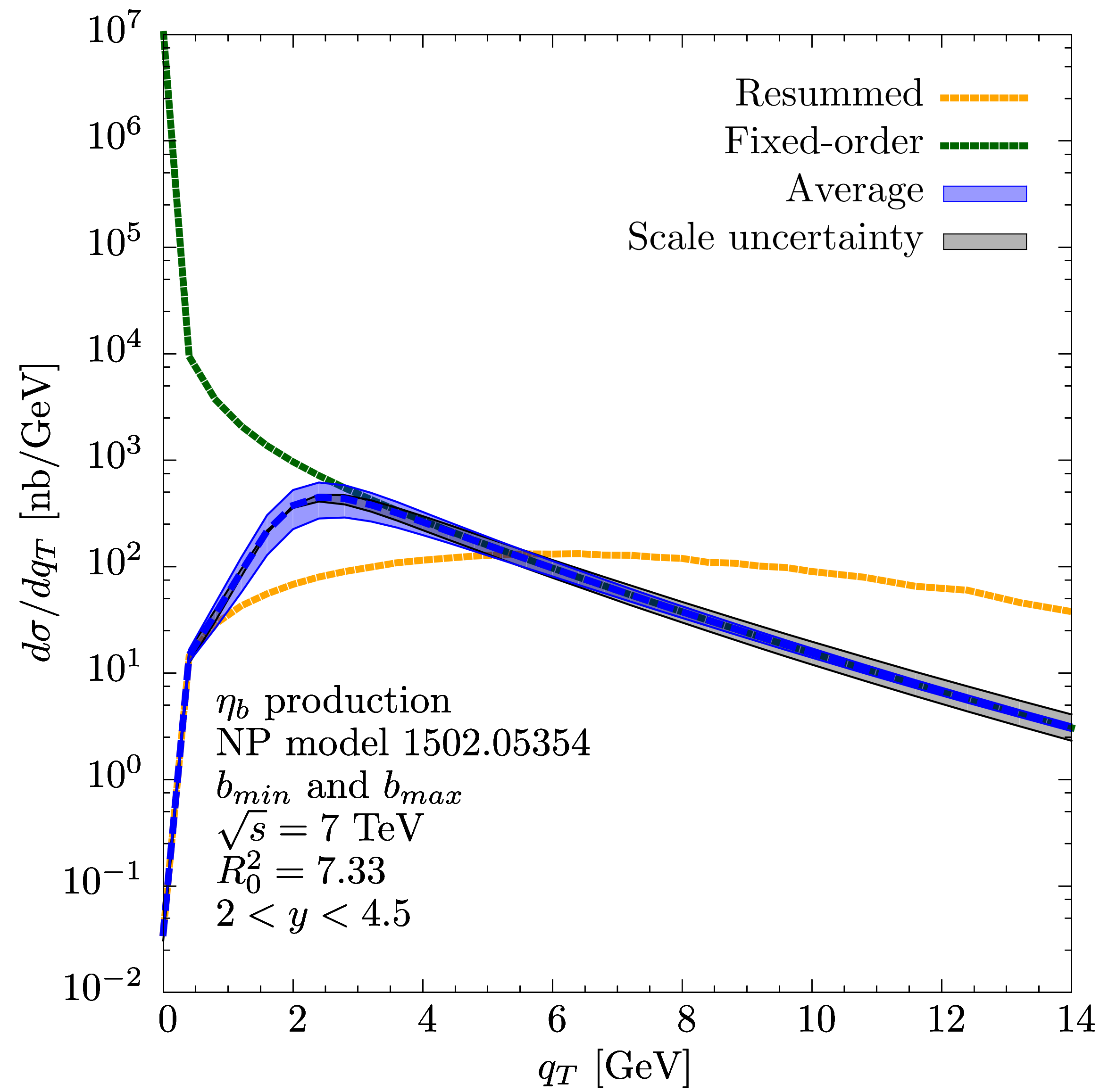
link with small-x physics

Matching for η_b production

What changes at higher Q ?



matching

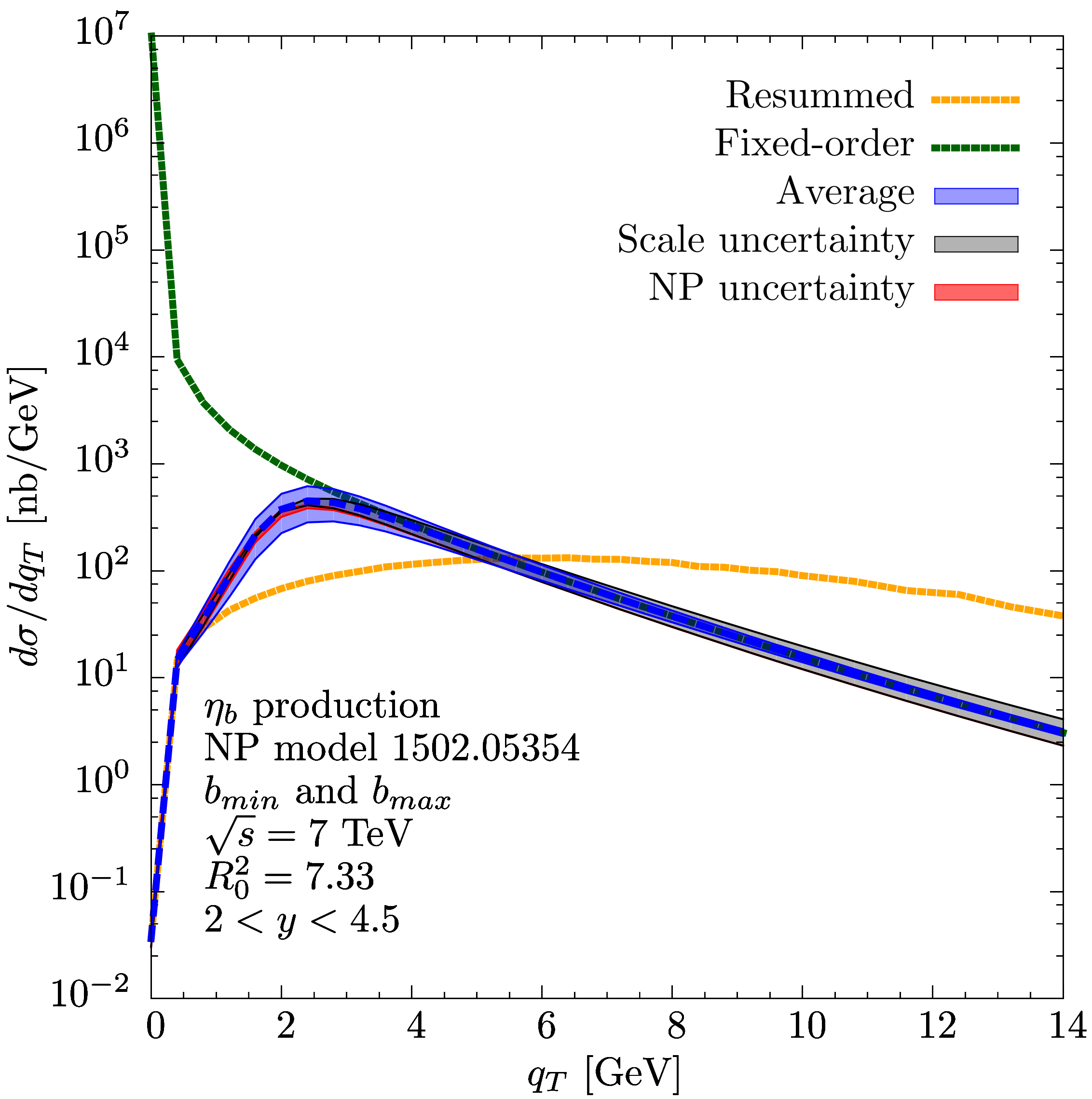


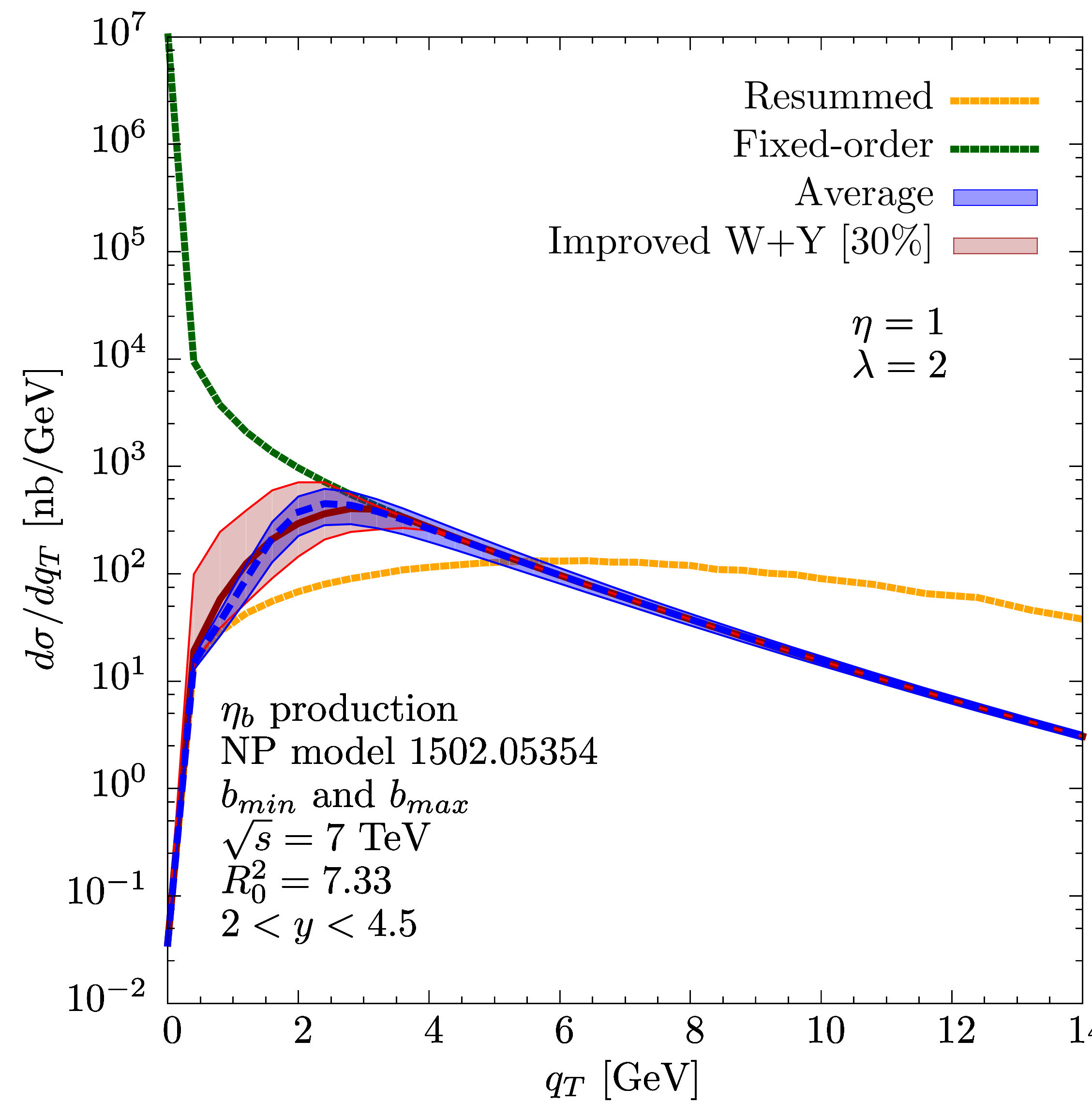
scale uncertainty shrink

W dominant till 1GeV, namely qT/Q < 1/9 (less important than for η_c)

What changes at higher Q ?

log-average vs improved W+Y





nonpert. uncertainty shrink

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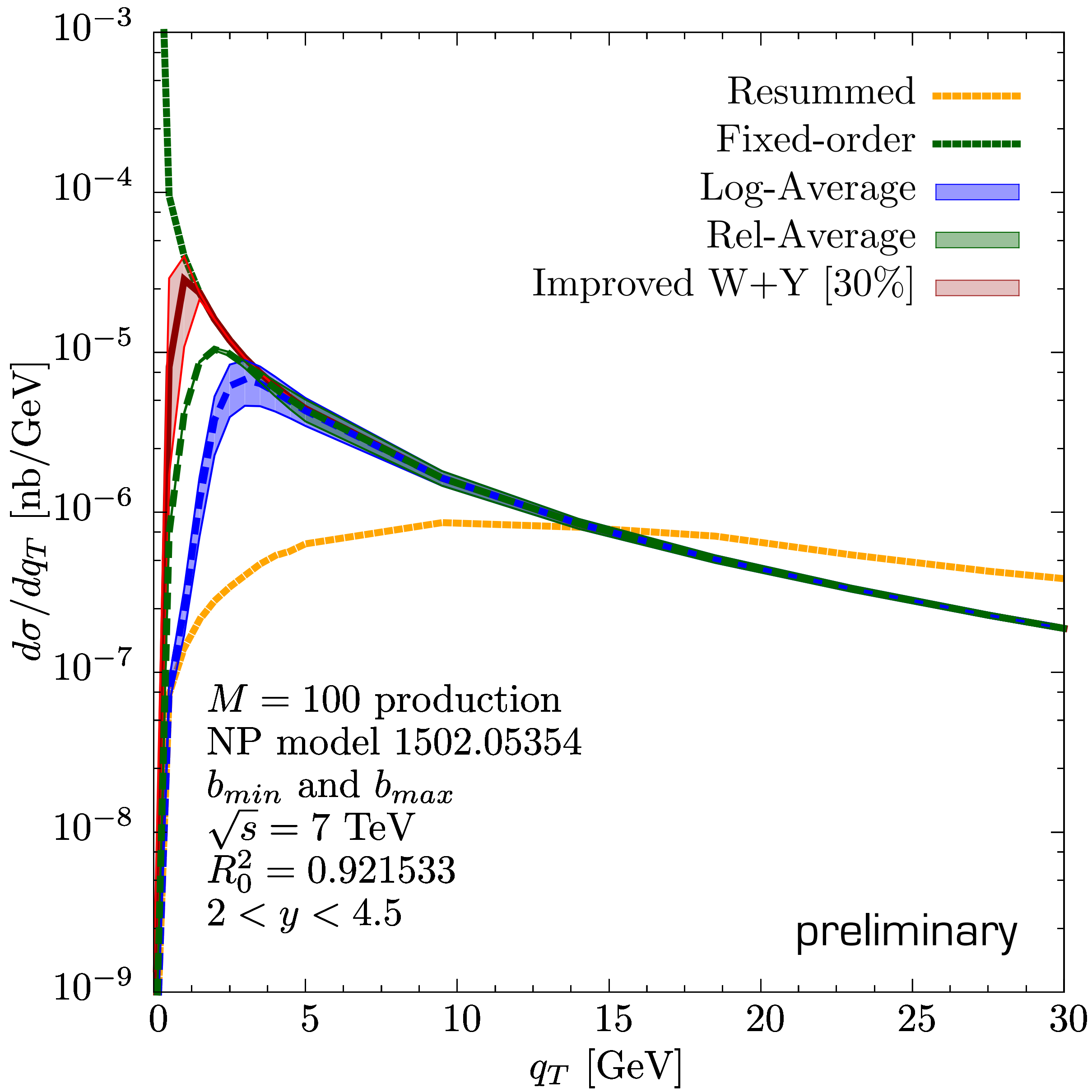
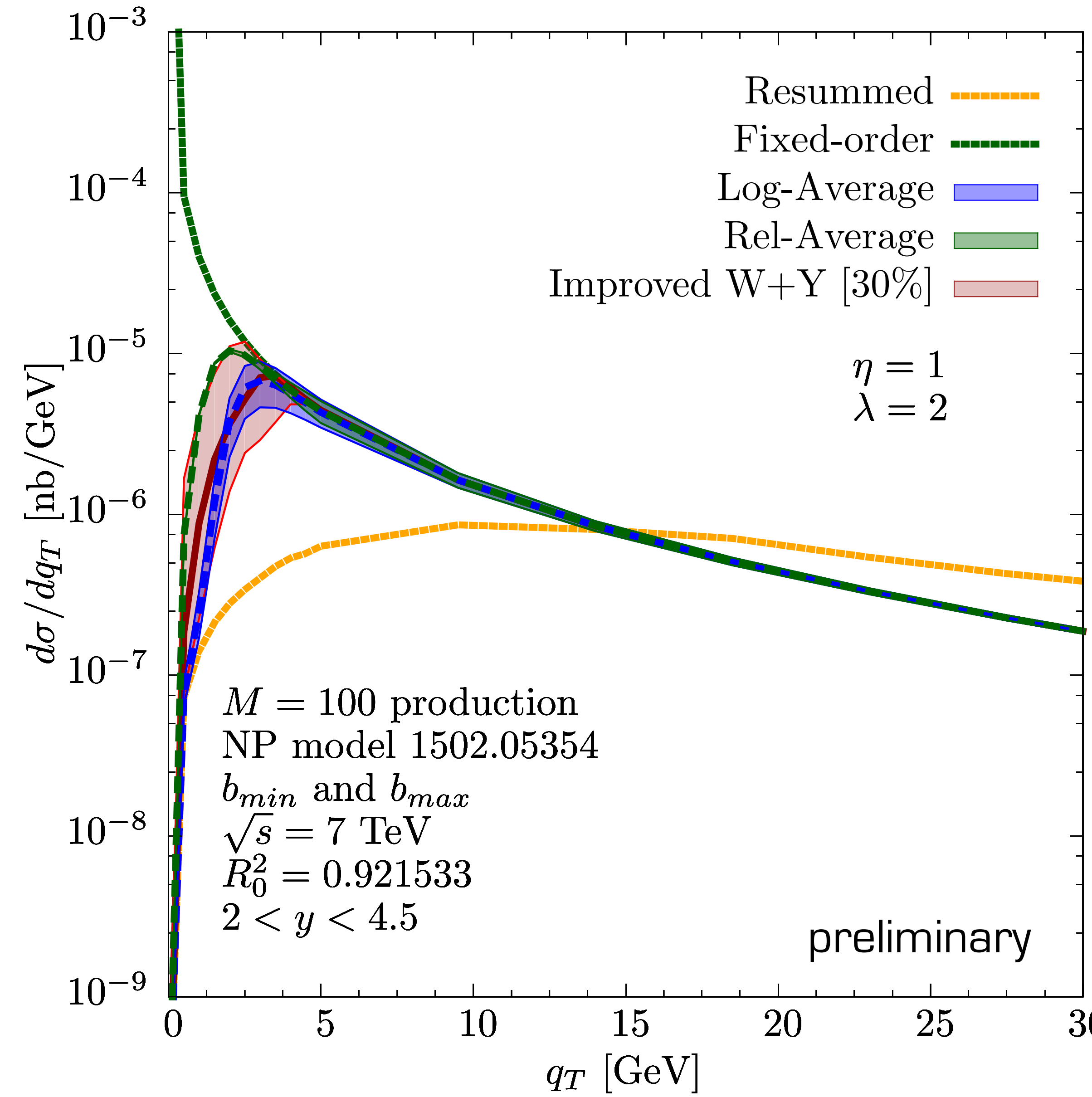
What changes at higher Q ?

Q = M = 100 GeV



log-average vs improved W+Y



$\eta_{b,c}$ production at LHC

References:
- AS, PhD thesis
- M.G. Echevarria, T. Kasemets, J.P. Lansberg, C. Pisano, AS - in preparation

TMD approach

$$\frac{d\sigma}{dy d^2 q_\perp} \sim \mathcal{O}^{q\bar{q}}(\eta_q)|C_H|^2 \Gamma^\dagger_{\mu\alpha}\Gamma_{\nu\beta}$$
$$\times \hat{\mathrm{FT}} \left[\underline{\tilde{G}^{\mu\nu}_{g/A}(x_A, b_T, S_A; \mu, \zeta_A)\; \tilde{G}^{\alpha\beta}_{g/B}(x_B, b_T, S_B; \mu, \zeta_B)} \right]$$
$$+ \mathcal{O}(q_T/M)$$

gluon correlators in IPS

medium/high q_T corrections

TMD factorization region

+ color singlet model

1) $|C_H|^2$ is the "hard part" : at this point still not known

2) NRQCD matrix element $\mathcal{O}^{q\bar{q}}(\eta_q) = |\langle 0|\chi^\dagger \psi(y)|\eta_q\rangle|^2 = \frac{N_c}{2\pi}|R_{nl}(0)|^2[1+\mathcal{O}(v^4)]$

3) Gamma structure fixed to reproduce the LO QCD result $\Gamma_{\mu\nu} = \frac{\alpha_s \pi}{3\sqrt{M}} \frac{2\sqrt{2}\epsilon^\perp_{\mu\nu}}{\sqrt{(d-2)(d-3)}}\sqrt{N_c^2-1}$

but no pole structure yet: go to next order!

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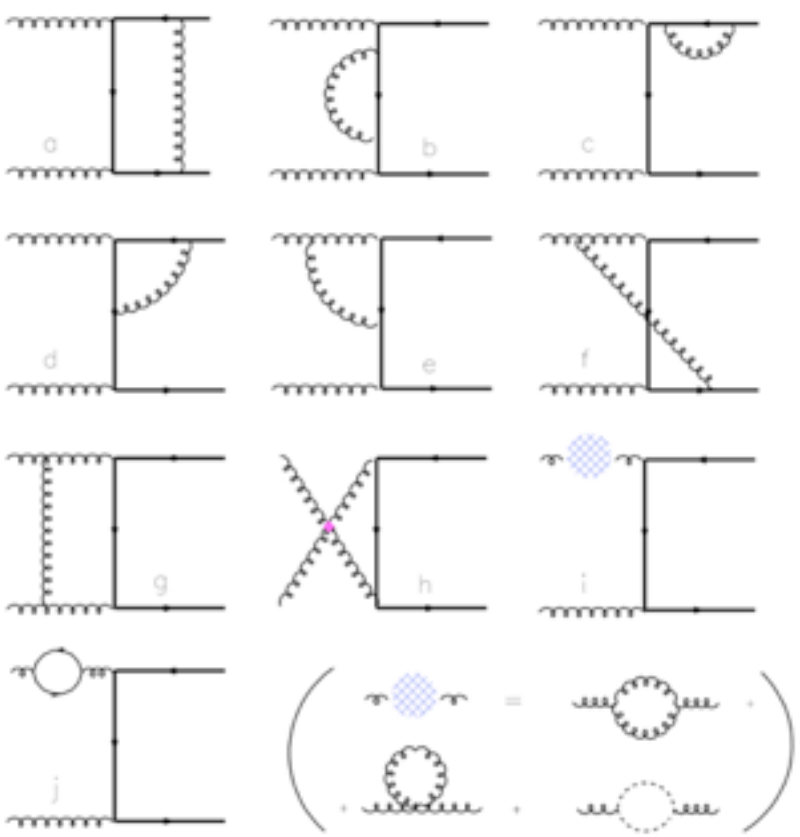
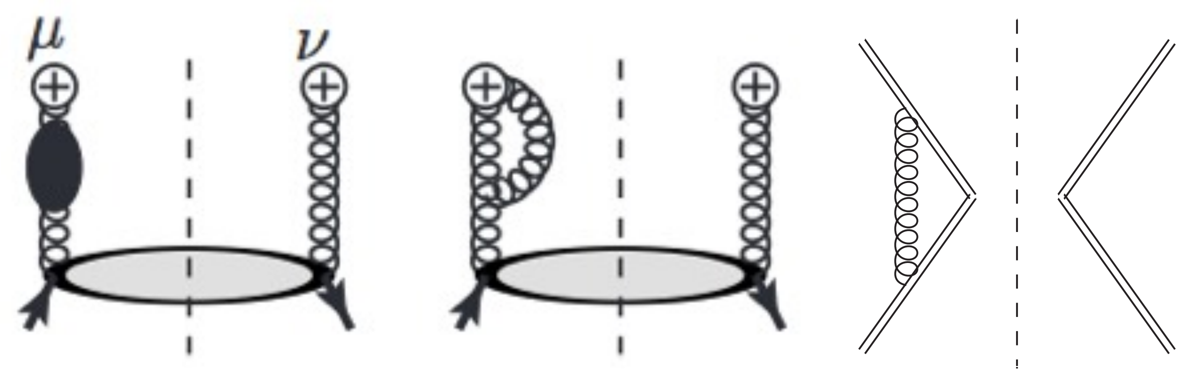
TMD approach



Philosophy : check if the structure of the IR divergencies is the same as in 'full' QCD. If yes, the factorized form works as QCD, namely factorization is "established"



$$\sigma^{\mathrm{virt},(1)} \quad \longleftrightarrow \quad \{\mathcal{H}\ \tilde{f}_1^{g/A}\,\tilde{f}_1^{g/B}\}^{(1)}_{\mathrm{virt}}$$

? same IR ?

no:
It does not reproduce the physical (=QCD) result

yes:
It reproduces the physical result and the hard part can be calculated by subtraction

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TMD approach

$$\frac{\sigma^v}{\sigma_{\text{Born}}}\Big|_{\text{ren}} = \frac{\alpha_s}{2\pi}\Bigg[-2\frac{C_A}{\epsilon_{\text{IR}}^2} - \frac{2}{\epsilon_{\text{IR}}}\left(\frac{\beta_0}{2} + C_A \ln\frac{\mu^2}{M^2}\right) + 2C_F\frac{\pi^2}{2v}$$
$$- C_A \ln^2\frac{\mu^2}{M^2} + 2C_A\left(1+\frac{\pi^2}{3}\right) + 2C_F\left(-5+\frac{\pi^2}{4}\right)\Bigg]$$

Coulomb singularity absorbed by NRQCD matrix element

renormalization takes care of UV

$$\tilde{f}_1^g = \frac{\alpha_s}{2\pi}\left[\frac{C_A}{\epsilon_{\text{UV}}^2} + \frac{1}{\epsilon_{\text{UV}}}\left(\frac{\beta_0}{2} + C_A\ln\frac{\mu^2}{\zeta_A}\right) - \frac{C_A}{\epsilon_{\text{IR}}^2} - \frac{1}{\epsilon_{\text{IR}}}\left(\frac{\beta_0}{2} + C_A\ln\frac{\mu^2}{\zeta_A}\right)\right]$$

X 2 TMDs

TMD approach

$$\frac{\sigma^v}{\sigma_{\rm Born}}\Big|_{\rm ren} = \frac{\alpha_s}{2\pi}\Bigg[-2\frac{C_A}{\epsilon_{\rm IR}^2} - \frac{2}{\epsilon_{\rm IR}}\left(\frac{\beta_0}{2} + C_A \ln\frac{\mu^2}{M^2}\right) + 2C_F\frac{\pi^2}{2v} - C_A \ln^2\frac{\mu^2}{M^2} + 2C_A\left(1+\frac{\pi^2}{3}\right) + 2C_F\left(-5+\frac{\pi^2}{4}\right)\Bigg]$$

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X 2 TMDs

Yes!

SCET correctly reproduces QCD at NLO

$$\mathcal{H} = \left[\ \sigma^{\rm virt,(1)} - \{\tilde{f}_1^{g/A}\tilde{f}_1^{g/B}\}^{(1)}_{\rm virt}\ \right]$$

on-shell renormalization scheme

$$\mathcal{H} = 1 + \frac{\alpha_s}{2\pi}\Bigg[- C_A \ln^2\frac{\mu^2}{M^2} + 2C_A\left(1+\frac{\pi^2}{3}\right) + 2C_F\left(-5+\frac{\pi^2}{4}\right)\Bigg]$$

see *Phys. Rev. D 70, 054014* (Maltoni&Polosa), Phys.Rev. D48 (1993) (Kuhn&Mirkes)

Useful

Re-factorization

OPE - matching coefficient

$$\tilde{T}_{g/A}(x, b_T; \mu, \zeta) = \sum_{j=q,\bar{q},g} \tilde{C}^T_{g/j}(x, b_T; \mu, \zeta) \otimes t_{j/A}(x; \mu) + \mathcal{O}(b_T \Lambda_{QCD})$$

TMD PDF

medium/high qT
Sudakov form factor
+ perturbative tail

PDF

intrinsic low q_T
(model)

Re-factorization

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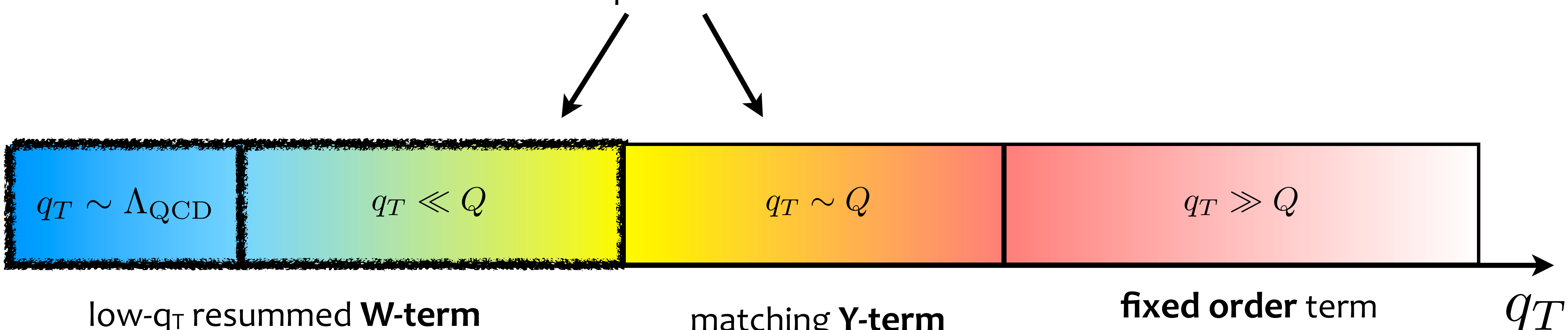

low-q_T resummed **W-term**
TMD factorization/evolution

matching **Y-term**

fixed order term
collinear factorization

transverse momentum spectrum of physical observables

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